APPENDIX B
Prerequisites

Appendix B.1 The Cartesian Plane

- You should be able to plot points.
- You should know that the distance between (x_1, y_1) and (x_2, y_2) in the plane is
  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- You should know that the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is
  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- You should know the equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$.
- You should be able to translate points in the plane.

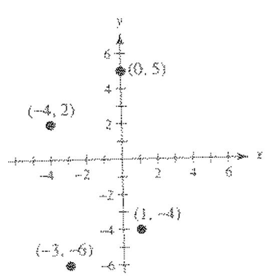
Vocabulary Check

1. (a) iii (b) vi (c) i (d) iv (e) v (f) ii
2. Cartesian
3. Distance Formula
4. Midpoint Formula
5. $(x - h)^2 + (y - k)^2 = r^2$, center, radius

1. A: $(2, 6)$, B: $(-6, -2)$, C: $(4, -4)$, D: $(-3, 2)$

2. A: $\left(\frac{3}{2}, -4\right)$; B: $(0, -2)$; C: $\left(-3, \frac{5}{2}\right)$; D: $(-6, 0)$

3.



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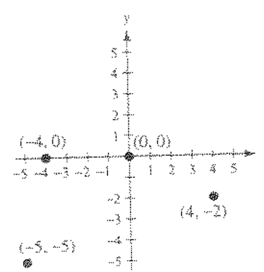

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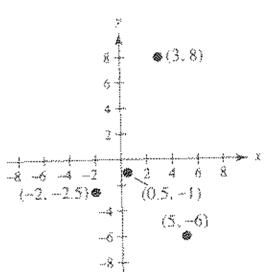

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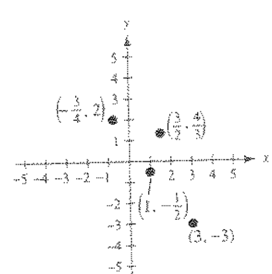


7. $(-5, 4)$

8. $(2, -3)$

9. $(-6, -6)$

10. $(-10, 0)$

11. $x > 0 \implies$ The point lies in Quadrant I or in Quadrant IV.

$y < 0 \implies$ The point lies in Quadrant III or in Quadrant IV.

$x > 0$ and $y < 0 \implies (x, y)$ lies in Quadrant IV.

12. If $x < 0$ and $y < 0$ then (x, y) is in Quadrant III.

13. $x = -4 \implies x$ is negative $\implies$ The point lies in Quadrant II or in Quadrant III.

$y > 0 \implies$ The point lies in Quadrant I or Quadrant II.

$x = -4$ and $y > 0 \implies (x, y)$ lies in Quadrant II.

14. If $x > 2$ and $y = 3$ then $(x, 3)$ is in Quadrant I.

15. $y < -5 \implies y$ is negative $\implies$ The point lies in either Quadrant III or Quadrant IV.

16. If $x > 4$ then (x, y) is in Quadrants I or IV.

17. If $-y > 0$, then $y < 0$.

$x < 0 \implies$ The point lies in Quadrant II or in Quadrant III.

$y < 0 \implies$ The point lies in Quadrant III or in Quadrant IV.

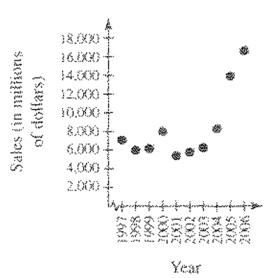
$x < 0$ and $y < 0 \implies (x, y)$ lies in Quadrant III.

18. If $(-x, y)$ is in Quadrant IV, then (x, y) must be in Quadrant III.

19. If $xy > 0$, then either x and y are both positive, or both negative. Hence, (x, y) lies in either Quadrant I or Quadrant III.

20. If $xy < 0$, then x and y have opposite signs. This happens in Quadrants II and IV.

21.



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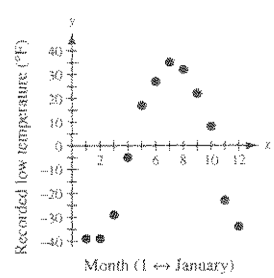


23. $(6, -3), (6, 5)$

$d = \sqrt{(6-6)^2 + (5-(-3))^2} = \sqrt{64} = 8$

24. $d = |8 - 1| = 7$

25. $(-3, -1), (2, -1)$

$d = \sqrt{(2-(-3))^2 + (-1-(-1))^2} = \sqrt{25} = 5$

26. $d = |6 - (-4)| = |6 + 4| = 10$

27. $d = \sqrt{(3-(-2))^2 + (-6-6)^2} = \sqrt{5^2 + (-12)^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$

28. $d = \sqrt{(0-8)^2 + (20-5)^2} = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17$

29. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$

$$d = \sqrt{\left(\frac{1}{2} - 2\right)^2 + \left(\frac{4}{3} + 1\right)^2}$$
$$= \sqrt{\frac{9}{4} + \frac{49}{9}}$$
$$= \sqrt{\frac{277}{36}} = \frac{\sqrt{277}}{6} \approx 2.77$$

30. $d = \sqrt{\left(-\frac{2}{3} + 1\right)^2 + \left(3 - \frac{5}{4}\right)^2}$

$$= \sqrt{\frac{1}{9} + \frac{49}{16}}$$
$$= \sqrt{\frac{457}{144}} = \frac{\sqrt{457}}{12} \approx 1.78$$

31. $(-4.2, 3.1), (-12.5, 4.8)$

$$d = \sqrt{(-4.2 + 12.5)^2 + (3.1 - 4.8)^2}$$
$$= \sqrt{68.89 + 2.89}$$
$$= \sqrt{71.78} \approx 8.47$$

32. $d = \sqrt{(9.5 + 3.9)^2 + (-2.6 - 8.2)^2}$

$$= \sqrt{179.56 + 116.64}$$
$$= \sqrt{296.2} \approx 17.21$$

33. (a) The distance between $(0, 2)$ and $(4, 2)$ is 4.

The distance between $(4, 2)$ and $(4, 5)$ is 3.

The distance between $(0, 2)$ and $(4, 5)$ is

$$\sqrt{(4 - 0)^2 + (5 - 2)^2} = \sqrt{16 + 9}$$
$$= \sqrt{25} = 5.$$

(b) $4^2 + 3^2 = 16 + 9 = 25 = 5^2$

34. (a) $(1, 0), (13, 5)$

$$d = \sqrt{(13 - 1)^2 + (5 - 0)^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= \sqrt{169} = 13$$

$(13, 5), (13, 0)$

$$d = |5 - 0| = |5| = 5$$

$(1, 0), (13, 0)$

$$d = |1 - 13| = |-12| = 12$$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

35. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9}$$
$$= \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

36. (a) $(1, 5), (5, -2)$

$$d = \sqrt{(1 - 5)^2 + (5 - (-2))^2}$$
$$= \sqrt{(-4)^2 + (7)^2}$$
$$= \sqrt{16 + 49} = \sqrt{65}$$

$(1, 5), (1, -2)$

$$d = |5 - (-2)| = |5 + 2| = |7| = 7$$

$(1, -2), (5, -2)$

$$d = |1 - 5| = |-4| = 4$$

(b) $4^2 + 7^2 = 65 = \left(\sqrt{65}\right)^2$

37. Find distances between pairs of points.

$$d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{5}$$
$$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{50}$$
$$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{45}$$
$$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$$

Because $d_1^2 + d_3^2 = d_2^2$, the triangle is a right triangle.

38. Find the distances between pairs of points.

$$d_1 = \sqrt{(3 - (-1))^2 + (5 - 3)^2}$$
$$= \sqrt{16 + 4} = \sqrt{20}$$
$$d_2 = \sqrt{(5 - 3)^2 + (1 - 5)^2}$$
$$= \sqrt{4 + 16} = \sqrt{20}$$
$$d_3 = \sqrt{(5 - (-1))^2 + (1 - 3)^2}$$
$$= \sqrt{36 + 4} = \sqrt{40}$$

Because $d_1^2 + d_2^2 = d_3^2$, the triangle is a right triangle.

39. $d_1 = \sqrt{(1-3)^2 + (-3-2)^2} = \sqrt{4+25} = \sqrt{29}$

$d_2 = \sqrt{(3+2)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$

$d_3 = \sqrt{(1+2)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$

$d_1 = d_2$. Triangle is isosceles.

40. Find the distances between pairs of points.

$d_1 = \sqrt{(4-2)^2 + (9-3)^2}$

$= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$d_2 = \sqrt{(-2-4)^2 + (7-9)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$d_3 = \sqrt{(-2-2)^2 + (7-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Because $d_1 = d_2$, the triangle is isosceles.

41. Find distances between pairs of points.

$d_1 = \sqrt{(0-2)^2 + (9-5)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$d_2 = \sqrt{(-2-0)^2 + (0-9)^2} = \sqrt{4+81} = \sqrt{85}$

$d_3 = \sqrt{(0-(-2))^2 + (-4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$d_4 = \sqrt{(0-2)^2 + (-4-5)^2} = \sqrt{4+81} = \sqrt{85}$

Opposite sides have equal lengths of $2\sqrt{5}$ and $\sqrt{85}$, so the figure is a parallelogram.

42. $d_1 = \sqrt{(0-3)^2 + (1-7)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

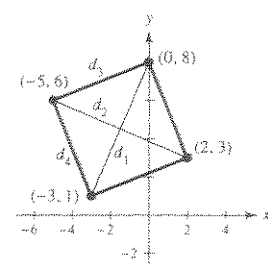
$d_2 = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10}$

$d_3 = \sqrt{(4-1)^2 + (4+2)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$d_4 = \sqrt{(0-1)^2 + (1+2)^2} = \sqrt{1+9} = \sqrt{10}$

Opposite sides have equal lengths of $3\sqrt{5}$ and $\sqrt{10}$. The figure is a parallelogram.

43. First show that the diagonals are equal in length.



$d_1 = \sqrt{(0-(-3))^2 + (8-1)^2} = \sqrt{9+49} = \sqrt{58}$

$d_2 = \sqrt{(2-(-5))^2 + (3-6)^2} = \sqrt{49+9} = \sqrt{58}$

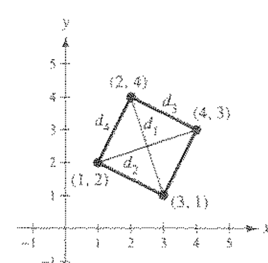
Now use the Pythagorean Theorem to verify that at least one angle is 90° (and, hence, they are all right angles).

$d_3 = \sqrt{(0-(-5))^2 + (8-6)^2} = \sqrt{25+4} = \sqrt{29}$

$d_4 = \sqrt{(-3-(-5))^2 + (1-6)^2} = \sqrt{4+25} = \sqrt{29}$

Thus, $d_3^2 + d_4^2 = d_1^2$.

44. First show that the diagonals are equal in length.



$d_1 = \sqrt{(3-2)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10}$

$d_2 = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$

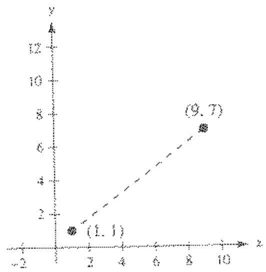
Now use the Pythagorean Theorem to verify that at least one angle is 90°.

$d_3 = \sqrt{(4-2)^2 + (3-4)^2} = \sqrt{4+1} = \sqrt{5}$

$d_4 = \sqrt{(2-1)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$

Thus, $d_3^2 + d_4^2 = d_1^2$.

45. (a)

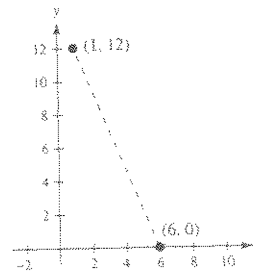


(b) $d = \sqrt{(9-1)^2 + (7-1)^2}$

$= \sqrt{64 + 36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

46. (a)

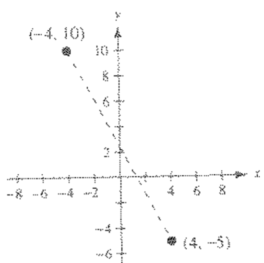


(b) $d = \sqrt{(1-6)^2 + (12-0)^2}$

$= \sqrt{25 + 144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

47. (a)

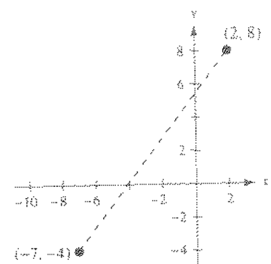


(b) $d = \sqrt{(4+4)^2 + (-5-10)^2}$

$= \sqrt{64 + 225} = 17$

(c) $\left(\dfrac{4-4}{2}, \dfrac{-5+10}{2}\right) = \left(0, \dfrac{5}{2}\right)$

48. (a)

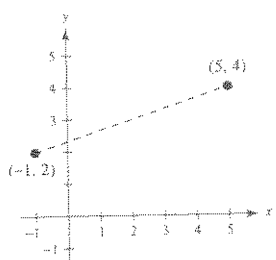


(b) $d = \sqrt{(-7-2)^2 + (-4-8)^2}$

$= \sqrt{81 + 144} = 15$

(c) $\left(\dfrac{-7+2}{2}, \dfrac{-4+8}{2}\right) = \left(-\dfrac{5}{2}, 2\right)$

49. (a)

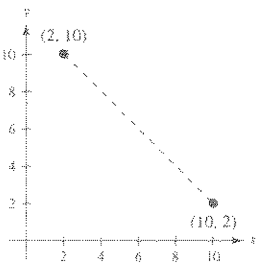


(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$

$= \sqrt{36 + 4} = \sqrt{40} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

50. (a)

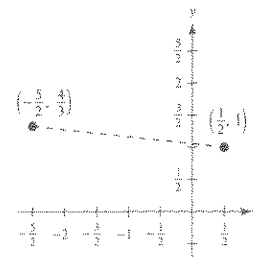


(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

51. (a)

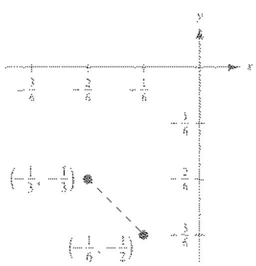


(b) $d = \sqrt{\left(\dfrac{1}{2}+\dfrac{5}{2}\right)^2 + \left(1-\dfrac{4}{3}\right)^2}$

$d = \sqrt{9+\dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-\frac{5}{2}+\frac{1}{2}}{2}, \dfrac{\frac{4}{3}+1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

52. (a)

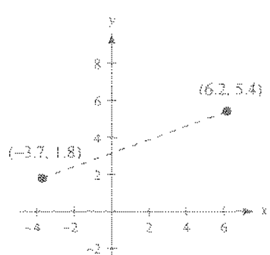


(b) $d = \sqrt{\left(-\dfrac{1}{3}+\dfrac{1}{6}\right)^2 + \left(-\dfrac{1}{3}+\dfrac{1}{2}\right)^2} = \sqrt{\dfrac{1}{36}+\dfrac{1}{36}} = \dfrac{\sqrt{2}}{6}$

(c) $\left(\dfrac{(-1/3)-(1/6)}{2}, \dfrac{(-1/3)-(1/2)}{2}\right) = \left(\dfrac{-1/2}{2}, \dfrac{-5/6}{2}\right)$

$= \left(-\dfrac{1}{4}, -\dfrac{5}{12}\right)$

53. (a)

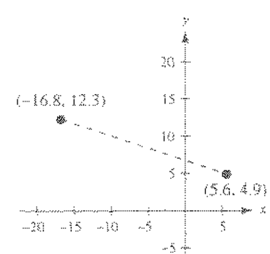


(b) $d = \sqrt{(6.2+3.7)^2 + (5.4-1.8)^2}$

$= \sqrt{98.01 + 12.96} = \sqrt{110.97}$

(c) $\left(\dfrac{6.2-3.7}{2}, \dfrac{5.4+1.8}{2}\right) = (1.25, 3.6)$

54. (a)



(b) $d = \sqrt{(-16.8-5.6)^2 + (12.3-4.9)^2}$

$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\dfrac{-16.8+5.6}{2}, \dfrac{12.3+4.9}{2}\right) = (-5.6, 8.6)$

55. Calculate the midpoint:

$\left(\dfrac{2000+2006}{2}, \dfrac{2237+3950}{2}\right) = (2003, 3093.5)$

The sales in 2003 are \$3093.5 million.

56. Calculate the midpoint:

$\left(\dfrac{2000+2006}{2}, \dfrac{945+1005}{2}\right) = (2003, 975)$

The sales in 2003 are \$975 million.

57. Since $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$

$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

(a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2(4) - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2(2) - (-5), 2(4) - 11) = (9, -3)$

58. (a) $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right) = \left(\dfrac{3(1) + 4}{4}, \dfrac{3(-2) - 1}{4}\right) = \left(\dfrac{7}{4}, \dfrac{-7}{4}\right)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 4}{2}, \frac{-2 - 1}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{1 + 3(4)}{4}, \frac{-2 + 3(-1)}{4}\right) = \left(\frac{13}{4}, \frac{-5}{4}\right)$$

(b) $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right) = \left(\dfrac{3(-2) + 0}{4}, \dfrac{3(-3) + 0}{4}\right) = \left(\dfrac{-3}{2}, \dfrac{-9}{4}\right)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{-3 + 0}{2}\right) = \left(-1, \frac{-3}{2}\right)$$

$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{-2 + 0}{4}, \frac{-3 + 0}{4}\right) = \left(\frac{-1}{2}, \frac{-3}{4}\right)$$

59. $(x - 0)^2 + (y - 0)^2 = 3^2$

$$x^2 + y^2 = 9$$

60. $(x - 0)^2 + (y - 0)^2 = 6^2$

$$x^2 + y^2 = 36$$

61. $(x - 2)^2 + (y + 1)^2 = 4^2$

$$(x - 2)^2 + (y + 1)^2 = 16$$

62. $(x - 0)^2 + \left(y - \frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2$

$$x^2 + \left(y - \tfrac{1}{3}\right)^2 = \tfrac{1}{9}$$

63. $(x + 1)^2 + (y - 2)^2 = r^2$

$$(0 + 1)^2 + (0 - 2)^2 = r^2 \implies r^2 = 5$$

$$(x + 1)^2 + (y - 2)^2 = 5$$

64. $r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2} = \sqrt{16 + 9} = 5$

$$(x - 3)^2 + (y + 2)^2 = 5^2 = 25$$

65. $r = \dfrac{1}{2}\sqrt{(6 - 0)^2 + (8 - 0)^2} = \dfrac{1}{2}\sqrt{100} = 5$

Center: $\left(\dfrac{0 + 6}{2}, \dfrac{0 + 8}{2}\right) = (3, 4)$

$$(x - 3)^2 + (y - 4)^2 = 25$$

66. Center: $\left(\dfrac{-4 + 4}{2}, \dfrac{-1 + 1}{2}\right) = (0, 0)$

$$r = \sqrt{(4 - 0)^2 + (1 - 0)^2} = \sqrt{17}$$

$$x^2 + y^2 = 17$$

67. Because the circle is tangent to the x-axis, the radius is 1.

$$(x + 2)^2 + (y - 1)^2 = 1$$

68. Because the circle is tangent to the y-axis, the radius is 3.

$$(x - 3)^2 + (y + 2)^2 = 9$$

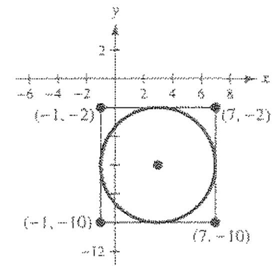
69. The center is the midpoint of one of the diagonals of the square.

Center: $\left(\dfrac{7+(-1)}{2}, \dfrac{-2+(-10)}{2}\right) = (3, -6)$

The radius is one half the length of a side of the square.

Radius: $\dfrac{1}{2}(7-(-1)) = 4$

Circle: $(x-3)^2 + (y+6)^2 = 16$



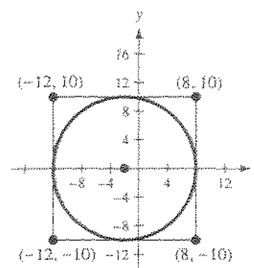
70. The center is the midpoint of one of the diagonals of the square.

Center: $\left(\dfrac{8+(-12)}{2}, \dfrac{10+(-10)}{2}\right) = (-2, 0)$

The radius is one half the length of a side of the square.

Radius: $\dfrac{1}{2}(8-(-12)) = 10$

Circle: $(x+2)^2 + y^2 = 100$



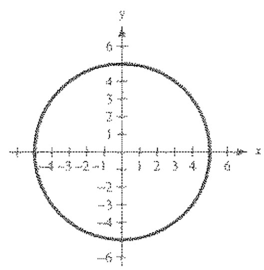
71. $(x-2)^2 + (y+1)^2 = 16$

72. $(x+3)^2 + (y-1)^2 = 25$

73. $x^2 + y^2 = 25$

Center: $(0, 0)$

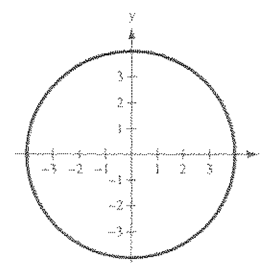
Radius: 5



74. $x^2 + y^2 = 16$

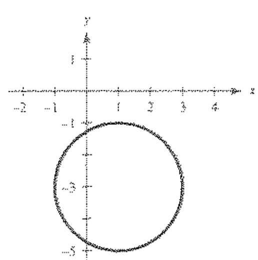
Center: $(0, 0)$

Radius: $\sqrt{16} = 4$



75. Center: $(1, -3)$

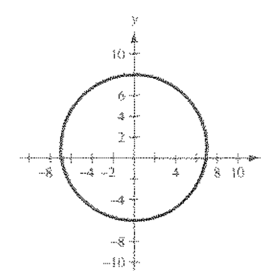
Radius: 2



76. $x^2 + (y-1)^2 = 49$

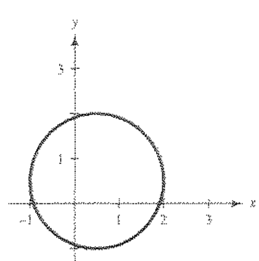
Center: $(0, 1)$

Radius: $\sqrt{49} = 7$



77. Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$

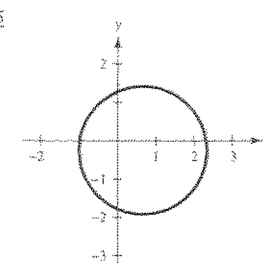
Radius: $\frac{3}{2}$



78. $\left(x-\frac{2}{3}\right)^2 + \left(y+\frac{1}{4}\right)^2 = \frac{25}{9}$

Center: $\left(\frac{2}{3}, -\frac{1}{4}\right)$

Radius: $\frac{5}{3}$



79. The x-coordinates are increased by 2, and the y-coordinates are increased by 5.

Old vertex	*Shifted vertex*
$(-1, -1)$	$(1, 4)$
$(-2, -4)$	$(0, 1)$
$(2, -3)$	$(4, 2)$

80. $(-3 + 6, 6 - 3) = (3, 3)$

$(-5 + 6, 3 - 3) = (1, 0)$

$(-3 + 6, 0 - 3) = (3, -3)$

$(-1 + 6, 3 - 3) = (5, 0)$

81.

Old vertex	*Shifted vertex*
$(0, 2)$	$(-1, 5)$
$(3, 5)$	$(2, 8)$
$(5, 2)$	$(4, 5)$
$(2, -1)$	$(1, 2)$

82. $(1 - 3, -1 - 2) = (-2, -3)$

$(3 - 3, 2 - 2), = (0, 0)$

$(1 - 3, -2 - 2) = (-2, -4)$

83. The point $(65, 83)$ represents an entrance exam score of 65.

84. No, there are many variables that will affect the final exam score.

85. (a) Sample answer: The number of artists inducted each year seems to be nearly steady except for the first few years.
Estimate: Between 5 and 7 new members

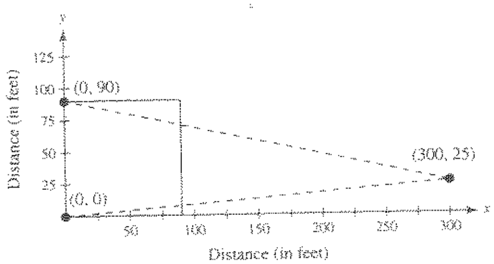
(b) Sample answer: The Rock and Roll Hall of Fame was opened in 1986.

86. Let $(0, 0)$ represent the point of departure, Naples, and $(120, 150)$ represent the destination, Rome.

$$d = \sqrt{(120 - 0)^2 + (150 - 0)^2}$$
$$= \sqrt{36{,}900} \approx 192.1 \text{ km}$$

87. $d = \sqrt{(45 - 10)^2 + (40 - 15)^2} = \sqrt{35^2 + 25^2} = \sqrt{1850} = 5\sqrt{74} \approx 43$ yards

88.



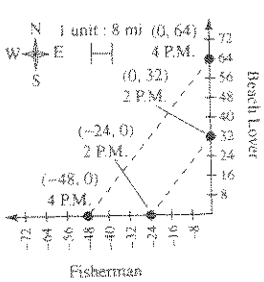
Distance from $(300, 25)$ to home plate:

$$d_1 = \sqrt{(300 - 0)^2 + (25 - 0)^2}$$
$$= \sqrt{90{,}625} \approx 301.0 \text{ feet}$$

Distance from $(300, 25)$ to third base:

$$d_2 = \sqrt{(300 - 0)^2 + (25 - 90)^2}$$
$$= \sqrt{94{,}225} \approx 307.0 \text{ feet}$$

89. (a)



(b) Distance at 2 P.M.:

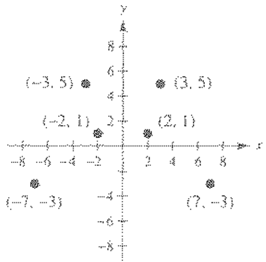
$$\sqrt{(-24 - 0)^2 + (0 - 32)^2} = \sqrt{1600} = 40 \text{ miles}$$

Distance at 4 P.M.:

$$\sqrt{(-48 - 0)^2 + (0 - 64)^2} = \sqrt{6400} = 80 \text{ miles}$$

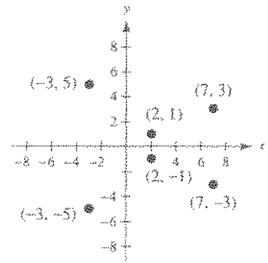
Yes, the yachts are twice as far from each other.

90. (a)



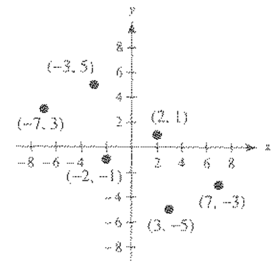
The points are reflected through the y-axis.

(b)



The points are reflected through the x-axis.

(c)



The points are rotated 180° about the origin (reflected through the origin).

91. Find the distances between pairs of points.

$$d_1 = \sqrt{(2 + 2\sqrt{3} - 2)^2 + (0 - 6)^2} = \sqrt{12 + 36} = \sqrt{48} = 4\sqrt{3}$$
$$d_2 = \sqrt{((2 + 2\sqrt{3}) - (2 - 2\sqrt{3}))^2 + (0 - 0)^2} = 4\sqrt{3}$$
$$d_3 = \sqrt{(2 - 2\sqrt{3} - 2)^2 + (0 - 6)^2} = \sqrt{12 + 36} = 4\sqrt{3}$$

Because $d_1 = d_2 = d_3$, the triangle is equilateral.

92. $$d_1 = \sqrt{(4 - (-2))^2 + (7 - (-1))^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$
$$d_2 = \sqrt{(4 - 2)^2 + (7 - (-4))^2} = \sqrt{4 + 121} = \sqrt{125} = 5\sqrt{5}$$
$$d_3 = \sqrt{(2 - (-2))^2 + (-4 - (-1))^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$
$$d_1^2 + d_3^2 = 100 + 25 = 125 = d_2^2$$

By the Pythagoran Theorem, the triangle is a right triangle.

93. False. It would be sufficient to use the midpoint formula 15 times.

94. True. The side joining $(-8, 4)$ and $(2, 11)$ has length $\sqrt{(-8 - 2)^2 + (4 - 11)^2} = \sqrt{149}$.

The side joining $(2, 11)$ and $(-5, 1)$ has length $\sqrt{(2 + 5)^2 + (11 - 1)^2} = \sqrt{149}$.

95. False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$ and $(0, -6)$.

96. The y-coordinate of a point on the x-axis is 0. The x-coordinate of a point on the y-axis is 0.

97. No, the scales can be different. The scales depend on the magnitude of the coordinates. See Figure P.13.

Appendix B.2 Graphs of Equations

- You should be able to use the point-plotting method of graphing.
- You should be able to find x- and y-intercepts.
  (a) To find the x-intercepts, let $y = 0$ and solve for x.
  (b) To find the y-intercepts, let $x = 0$ and solve for y.
- You should know how to graph an equation with a graphing utility. You should be able to determine an appropriate viewing rectangle.
- You should be able to use the zoom and trace features of a graphing utility.

Vocabulary Check

1. solution point
2. graph
3. intercepts

1. $y = \sqrt{x + 4}$

(a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$2 = 2$ ✓

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$3 = \sqrt{9}$ ✓

Yes, the point *is* on the graph.

2. $y = x^2 - 3x + 2$

(a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

3. $y = 4 - |x - 2|$

(a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \neq 4 - 1$

No, the point *is not* on the graph.

(b) $(1.2, 3.2)$: $3.2 \stackrel{?}{=} 4 - |1.2 - 2|$

$3.2 \stackrel{?}{=} 4 - |-0.8|$

$3.2 \stackrel{?}{=} 4 - 0.8$

$3.2 \stackrel{?}{=} 3.2$ ✓

Yes, the point *is* on the graph.

4. $2x - y - 3 = 0$

(a) $(1, 2)$: $2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point *is not* on the graph.

(b) $(1, -1)$: $2(1) - (-1) - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

5. $x^2 + y^2 = 20$

(a) $(3, -2)$: $3^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + 2^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

6. $y = \frac{1}{3}x^3 - 2x^2$

(a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$

$-\frac{16}{3} = -\frac{16}{3}$

Yes, the point is on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$

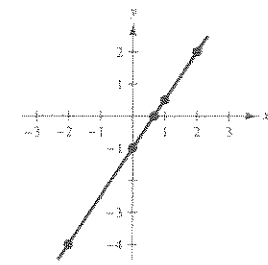
$-9 - 18 \stackrel{?}{=} 9$

$-27 \neq 9$

No, the point is not on the graph.

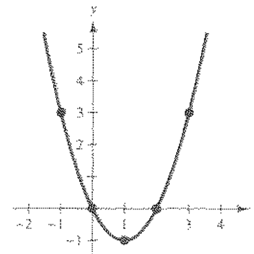
7. $y = \frac{3}{2}x - 1$

x	-2	0	$\frac{2}{3}$	1	2
y	-4	-1	0	$\frac{1}{2}$	2
Solution point	$(-2, -4)$	$(0, -1)$	$(\frac{3}{2}, 0)$	$(1, \frac{1}{2})$	$(2, 2)$



8. $y = x^2 - 2x$

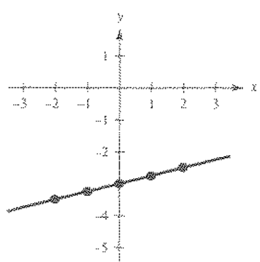
x	-1	0	1	2	3
y	3	0	-1	0	3
Solution point	$(-1, 3)$	$(0, 0)$	$(1, -1)$	$(2, 0)$	$(3, 3)$



9. (a) $y = \frac{1}{4}x - 3$

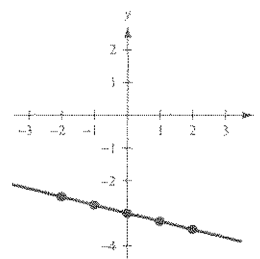
x	-2	-1	0	1	2
y	$-\frac{7}{2}$	$-\frac{13}{4}$	-3	$-\frac{11}{4}$	$-\frac{5}{2}$

(b)



(c) $y = -\frac{1}{4}x - 3$

x	-2	-1	0	1	2
y	$-\frac{5}{2}$	$-\frac{11}{4}$	-3	$-\frac{13}{4}$	$-\frac{7}{2}$

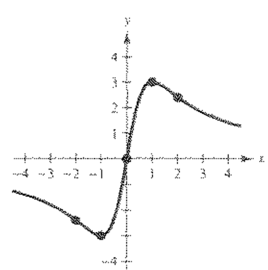


Both graphs are lines. The first graph rises to the right, whereas the second falls. Both pass through $(0, -3)$.

10. (a) $y = \dfrac{6x}{x^2 + 1}$

x	-2	-1	0	1	2
y	-2.4	-3	0	3	2.4

(b)



(c)

x	5	10	20	40
y	1.15	0.59	0.30	0.15

The y-values are approaching 0.

No, y cannot be negative for positive values of x because $6x > 0$ and $x^2 + 1 > 0$.

11. $y = 2x + 3$ has intercepts $(0, 3)$ and $\left(-\frac{3}{2}, 0\right)$.

Matches graph (e).

12. $y = 4 - x^2$ has intercepts $(0, 4)$, $(2, 0)$ and $(-2, 0)$.

Matches graph (f).

13. $y = x^2 - 2x$ has intercepts $(0, 0)$ and $(2, 0)$.

Matches graph (b).

14. $y = \sqrt{9 - x^2}$ has intercepts $(0, 3)$, $(-3, 0)$ and $(3, 0)$.

Matches graph (d).

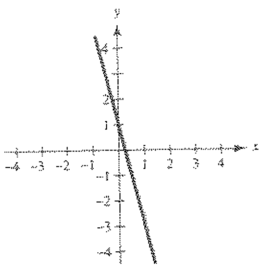
15. $y = 2\sqrt{x}$ has one intercept $(0, 0)$.

Matches graph (c).

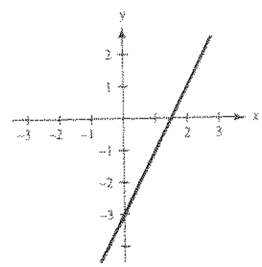
16. $y = |x| - 3$ has intercepts $(0, -3)$, $(3, 0)$ and $(-3, 0)$.

Matches graph (a).

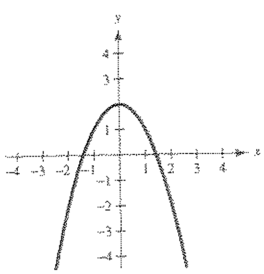
17. $y = -4x + 1$



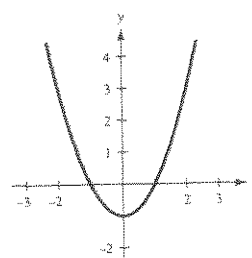
18. $y = 2x - 3$



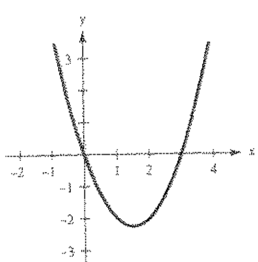
19. $y = 2 - x^2$



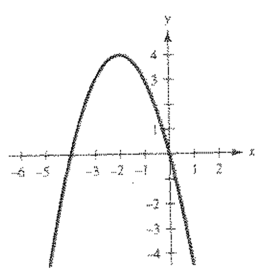
20. $y = x^2 - 1$



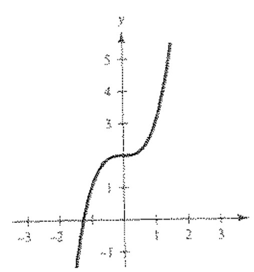
21. $y = x^2 - 3x$



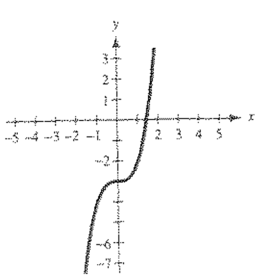
22. $y = -x^2 - 4x$



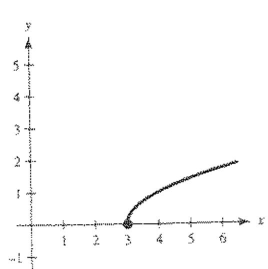
23. $y = x^3 + 2$



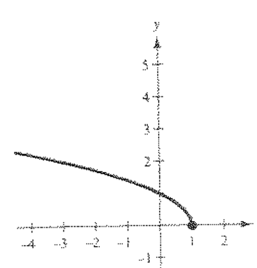
24. $y = x^3 - 3$



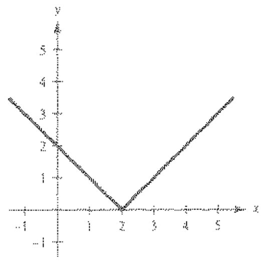
25. $y = \sqrt{x - 3}$



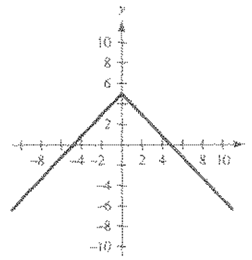
26. $y = \sqrt{1 - x}$



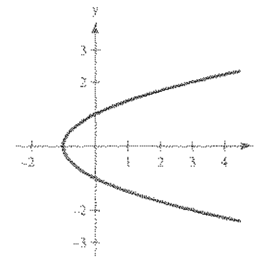
27. $y = |x - 2|$



28. $y = 5 - |x|$

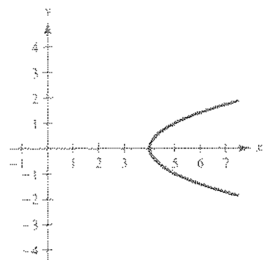


29. $x = y^2 - 1$



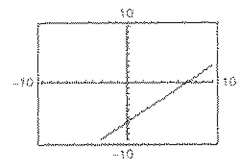
30. $x = y^2 + 4$

Intercept: $(4, 0)$



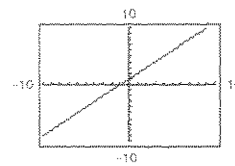
31. $y = x - 7$

Intercepts: $(0, -7)$, $(7, 0)$



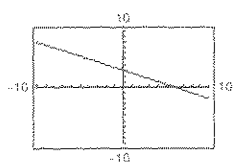
32. $y = x + 1$

Intercepts: $(-1, 0)$, $(0, 1)$



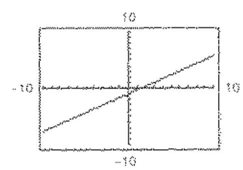
33. $y = 3 - \frac{1}{2}x$

Intercepts: $(6, 0)$, $(0, 3)$



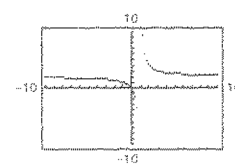
34. $y = \frac{2}{3}x - 1$

Intercepts: $(0, -1)$, $\left(\frac{3}{2}, 0\right)$

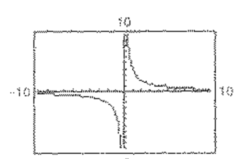


35. $y = \frac{2x}{x - 1}$

Intercepts: $(0, 0)$

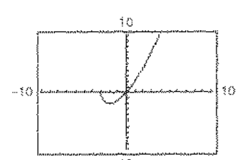


36. $y = \frac{4}{x}$



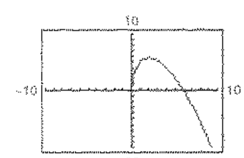
37. $y = x\sqrt{x + 3}$

Intercepts: $(0, 0)$, $(-3, 0)$



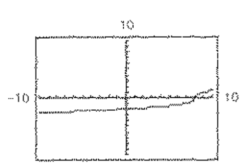
38. $y = (6 - x)\sqrt{x}$

Intercepts: $(0, 0)$, $(6, 0)$



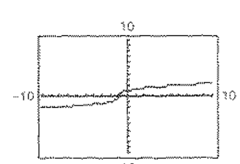
39. $y = \sqrt[3]{x - 8}$

Intercepts: $(8, 0)$, $(0, -2)$



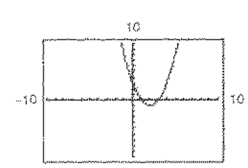
40. $y = \sqrt[3]{x + 1}$

Intercepts: $(-1, 0)$, $(0, 1)$



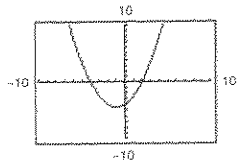
41. $y = x^2 - 4x + 3$

Intercepts: $(3, 0)$, $(1, 0)$, $(0, 3)$



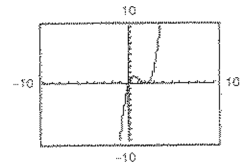
42. $y = \dfrac{x^2 + 2x - 8}{2}$

Intercepts:
$(2, 0), (-4, 0), (0, -4)$



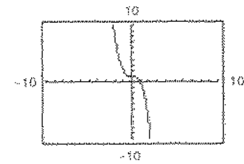
43. $y = x^2(x - 4) + 4x$
$= x^3 - 4x^2 + 4x$

Intercepts: $(0, 0), (2, 0)$

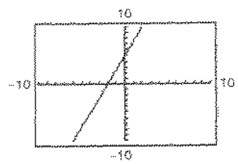


44. $y = 1 - x^3$

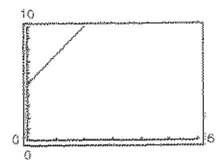
Intercepts: $(0, 1), (1, 0)$



45. $y = \frac{5}{2}x + 5$

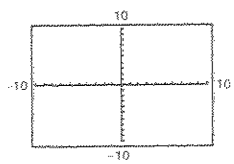


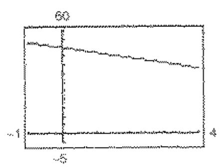
The first setting shows the line and its intercepts. The first setting is better.



The second setting does not show the x-intercept $(-2, 0)$.

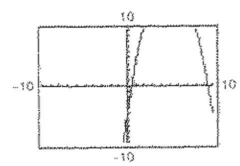
46. $y = -3x + 50$

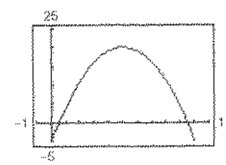




The specified setting gives a more complete graph. (The y-intercept is visible.)

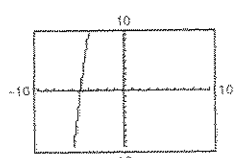
47. $y = -x^2 + 10x - 5$

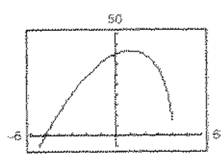




The second viewing window is better because it shows more of the essential features of the function.

48. $y = 4(x + 5)\sqrt{4 - x}$





The specified setting gives a more complete graph.

49. $y = -10x + 50$

Range/Window

Xmin = -10
Xmax = 10
Xscl = 2
Ymin = -50
Ymax = 100
Yscl = 25

50. $y = 4x^2 - 25$

Range/Window

Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -30
Ymax = 10
Yscl = 5

51. $y = \sqrt{x + 2} - 1$

Range/Window

Xmin = -5
Xmax = 1
Xscl = 1
Ymin = -3
Ymax = 1
Yscl = 1

52. $y = x^3 - 3x^2 + 4$

Range/Window

Xmin = -3
Xmax = 5
Xscl = 1
Ymin = -3
Ymax = 5
Yscl = 1

53. $y = |x| + |x - 10|$

Range/Window

Xmin = -30
Xmax = 30
Xscl = 5
Ymin = -10
Ymax = 50
Yscl = 5

54. $y = 8\sqrt[3]{x - 6}$

Range/Window

Xmin = -40
Xmax = 40
Xscl = 10
Ymin = -40
Ymax = 40
Yscl = 10

55. $y_1 = \frac{1}{4}(x^2 - 8)$

$y_2 = \frac{1}{4}x^2 - 2$

The graphs are identical.

The Distributive Property is illustrated.

56. $y_1 = \frac{1}{2}x + (x + 1)$

$y_2 = \frac{3}{2}x + 1$

Graphing these with a graphing utility shows that their graphs are identical. The Associative Property of Addition is illustrated.

57. $y_1 = \frac{1}{5}[10(x^2 - 1)]$

$y_2 = 2(x^2 - 1)$

The graphs are identical.

The Associative Property of Multiplication is illustrated.

58. $y_1 = (x - 3) \cdot \dfrac{1}{x - 3}$

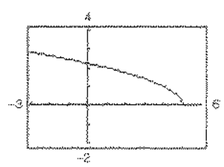
$y_2 = 1$

Graphing these with a graphing utility shows that their graphs are identical. The Multiplicative Inverse Property is illustrated (except for hole at $x = 3$ for y_1).

59. $y = \sqrt{5 - x}$

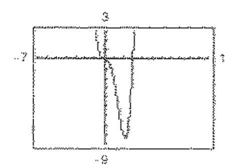
(a) $(2, y) \approx (2, 1.73)$

(b) $(x, 3) = (-4, 3)$



60. (a) $(2.25, -8.54)$

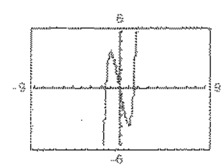
(b) $(-1.63, 20), (3.48, 20)$



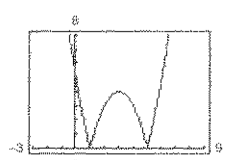
61. $y = x^5 - 5x$

(a) $(-0.5, y) \approx (-0.5, 2.47)$

(b) $(x, -4) = (1, -4)$ or $(x, -4) \approx (-1.65, -4)$



62.



(a) $(2, 3)$

(b) $(0.65, 1.5), (1.42, 1.5)$

$(4.58, 1.5), (5.35, 1.5)$

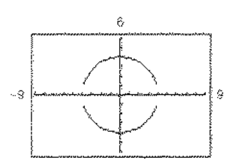
63. $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16 - x^2}$

Use $y_1 = \sqrt{16 - x^2}$

$y_2 = -\sqrt{16 - x^2}$



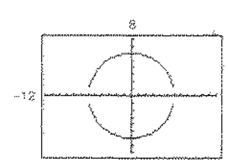
64. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \pm\sqrt{36 - x^2}$

Use $y_1 = \sqrt{36 - x^2}$

$y_2 = -\sqrt{36 - x^2}$

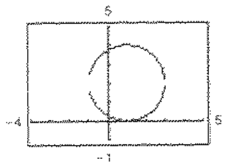


65. $(x-1)^2 + (y-2)^2 = 4$

$$(y-2)^2 = 4 - (x-1)^2$$
$$y - 2 = \pm\sqrt{4-(x-1)^2}$$
$$y = 2 \pm \sqrt{4-(x-1)^2}$$

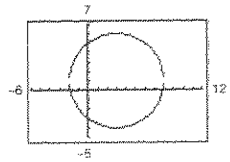
Use $y_1 = 2 + \sqrt{4-(x-1)^2}$

$y_2 = 2 - \sqrt{4-(x-1)^2}$



66. $y_1 = 1 + \sqrt{25-(x-3)^2}$

$y_2 = 1 - \sqrt{25-(x-3)^2}$



67. The center is in the first quadrant and the circle is tangent to the x-axis.
Matches (a).

68. The center is in the second quadrant and the circle intercepts the axes.
Matches (c).

69. $(x-1)^2 + (y-2)^2 = 25$

(a) $(1-1)^2 + (2-2)^2 = 0 \neq 25$ No

(b) $(-2-1)^2 + (6-2)^2 = 9 + 16 = 25$ Yes

(c) $(5-1)^2 + (-1-2)^2 = 16 + 9 = 25$ Yes

(d) $(0-1)^2 + \left(2 + 2\sqrt{6} - 2\right)^2 = 1 + 24 = 25$ Yes

70. $(x+2)^2 + (y-3)^2 = 25$

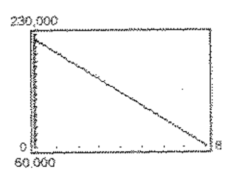
(a) $(-2+2)^2 + (3-3)^2 = 0 \neq 25$ No

(b) $(0+2)^2 + (0-3)^2 = 4 + 9 = 13 \neq 25$ No

(c) $(1+2)^2 + (-1-3)^2 = 9 + 16 = 25$ Yes

(d) $(-1+2)^2 + \left(3 - 2\sqrt{6} - 3\right)^2 = 1 + 24 = 25$ Yes

71. (a) $y = 225{,}000 - 20{,}000t\ ,\ 0 \le t \le 8$



Window

$X_{\min} = 0$

$X_{\max} = 8$

$X_{\text{scl}} = 1$

$Y_{\min} = 60{,}000$

$Y_{\max} = 230{,}000$

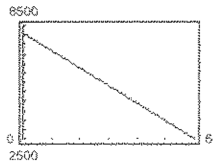
$Y_{\text{scl}} = 10{,}000$

(b) When $t = 5.8$, $y = 109{,}000$. Algebraically, $225{,}000 - 20{,}000(5.8) = \$109{,}000$.

(c) When $t = 2.35$, $y = 178{,}000$. Algebraically, $225{,}000 - 20{,}000(2.35) = \$178{,}000$.

72. (a)

Xmin = 0
Xmax = 6
Xscl = 1
Ymin = 2500
Ymax = 8500
Yscl = 500



(b) For $y = 5545.25$, $t = 2.75$. Algebraically,

$$8100 - 929t = 5545.25$$
$$2554.75 = 929t$$
$$t = 2.75 \text{ years.}$$

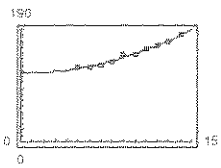
(c) For $t = 5.5$, $y = 2990.5$. Algebraically,
$y = 8100 - 929(5.5) = \$2990.5$.

73. (a) Model: $y = -0.0049t^3 + 0.443t^2 - 0.75t + 116.7, \quad 5 \le t \le 14$

t	5	6	7	8	9	10	11	12	13	14
Model	123.4	127.1	131.5	136.5	142.3	148.6	155.5	163.0	171.1	179.6

The model is a good fit.

(b)

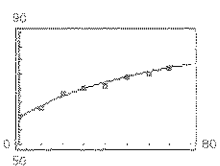


The model is a good fit.

(c) For 2008, $t = 18$ and $y \approx 218.2$ thousand dollars. For 2010, $t = 20$ and $y \approx 239.7$ thousand dollars. These values seem reasonable.

(d) For $y = 150$, $t \approx 10.2$, or 2000.

74. (a)



The model is a good fit.

(b) The y-intercept represents the life expectancy for the year 1930.

(c) $y = 73.2$ for $t \approx 45$, or 1975

(d) For 1948, $t = 18$ and $y \approx 66.5$ years.

$$\text{Algebraically, } y = \frac{59.617 + 1.18(18)}{1 + 0.012(18)}$$
$$= \frac{80.857}{1.216} \approx 66.5$$

(e) For 2010, $t = 80$ and $y \approx 78.6$ years.

75. (a)

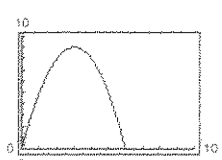
(b) Perimeter: $12 = 2x + 2w$

$$12 = 2(x + w)$$
$$6 = x + w$$

Thus, $w = 6 - x$.

Area: $xw = x(6 - x) \implies A = x(6 - x)$

(c)



(d) When $w = 4.9$, $x = 1.1$ and Area $= 5.39$ square meters.

Algebraically,
Area $= xw = (1.1)(4.9) = 5.39$ square meters.

(e) The maximum area corresponds to the highest point on the graph, which appears to be $(3, 9)$. Thus, $x = 3$ and $w = 3$, and the rectangle is a square.

76. Center $= \left(\frac{0+4}{2}, \frac{0-6}{2}\right) = (2, -3)$

Radius $= \frac{1}{2}\sqrt{(0-4)^2 + (0+6)^2}$

$= \frac{1}{2}\sqrt{16+36} = \frac{1}{2}\sqrt{52} = \sqrt{13}$

Circle: $(x-2)^2 + (y+3)^2 = 13$

77. False. $y = 1 - x^2$ has two x-intercepts, $(1, 0)$ and $(-1, 0)$. Also, $y = x^2 + 1$ has no x-intercepts.

78. False. The line $y = 0$ has an infinite number of x-intercepts.

79. Answers will vary.

80. Option 1: $w_1 = 3000 + 0.07x$

Option 2: $w_2 = 3400 + 0.05x$

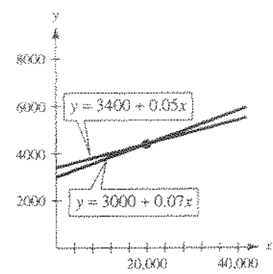
(x is amount of sales)

$w_1 = w_2$

$3000 + 0.07x = 3400 + 0.05x$

$0.02x = 400$

$x = 20{,}000$



If sales equal 20,000, the options are equivalent. For sales less than 20,000, choose option 2. For sales greater than 20,000, choose option 1.

81. Answers will vary. Sample answer: $y = 250x + 1000$ could represent the amount of money in someone's checking account after x months if they deposited an initial \$1000 and added \$250 per month.

82. Answers will vary. Sample answer: The equation could represent the amount of points for turning in a 10 point assignment x days late.

Appendix B.3 Solving Equations Algebraically and Graphically

- You should know how to solve linear equations: $ax + b = 0$.
- An identity is an equation whose solution consists of every real number in its domain.
- To solve an equation you can:
  (a) Add or subtract the same quantity from both sides.
  (b) Multiply or divide both sides by the same nonzero quantity.
- To solve an equation that can be simplified to a linear equation:
  (a) Remove all symbols of grouping and all fractions. (b) Combine like terms.
  (c) Solve by algebra. (d) Check the answer.
- A "solution" that does not satisfy the original equation is called an extraneous solution.
- You should be able to set up mathematical models to solve problems.

—CONTINUED—

Appendix B.3 —CONTINUED—

- You should be able to translate key words and phrases.
  - (a) Equality: Equals, equal to, is, are, was, will be, represents
  - (b) Addition: Sum, plus, greater, increased by, more than, exceeds, total of
  - (c) Subtraction: Difference, minus, less than, decreased by, subtracted from, reduced by, the remainder
  - (d) Multiplication: Product, multiplied by, twice, times, percent of
  - (e) Division: Quotient, divided by, ratio, per
  - (f) Consecutive: Next, subsequent
- You should know the following formulas:
  - (a) Perimeter:
    1. Square: $P = 4s$
    2. Rectangle: $P = 2L + 2W$
    3. Circle: $C = 2\pi r$
  - (b) Area:
    1. Square: $A = s^2$
    2. Rectangle: $A = LW$
    3. Circle: $A = \pi r^2$
    4. Triangle: $A = \left(\frac{1}{2}\right)bh$
  - (c) Volume
    1. Cube: $V = s^3$
    2. Rectangular solid: $V = LWH$
    3. Cylinder: $V = \pi r^2 h$
    4. Sphere: $V = \left(\frac{4}{3}\right)\pi r^3$
  - (d) Simple Interest: $I = Prt$
  - (e) Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
  - (f) Distance: $D = r \cdot t$
  - (g) Temperature: $F = \frac{9}{5}C + 32$
- You should be able to solve word problems. Study the examples in the text carefully.

1. $\frac{5}{2x} - \frac{4}{x} = 3$

(a) $\frac{5}{2(-1/2)} - \frac{4}{(-1/2)} \stackrel{?}{=} 3$

$3 = 3$

$x = -\frac{1}{2}$ *is* a solution.

(b) $\frac{5}{2(4)} - \frac{4}{4} \stackrel{?}{=} 3$

$-\frac{3}{8} \neq 3$

$x = 4$ *is not* a solution.

(c) $\frac{5}{2(0)} - \frac{4}{0}$ is undefined.

$x = 0$ *is not* a solution.

(d) $\frac{5}{2(1/4)} - \frac{4}{1/4} \stackrel{?}{=} 3$

$-6 \neq 3$

$x = \frac{1}{4}$ *is not* a solution.

2. $\frac{x}{2} + \frac{6x}{7} = \frac{19}{14}$

(a) $x = -2$

$$\frac{-2}{2} + \frac{6(-2)}{7} \stackrel{?}{=} \frac{19}{14}$$

$$\frac{-14 - 24}{14} \stackrel{?}{=} \frac{19}{14}$$

$$\frac{-38}{14} \stackrel{?}{=} \frac{19}{14}$$

$x = -2$ is not a solution.

(c) $x = \frac{1}{2}$

$$\frac{1/2}{2} + \frac{6(1/2)}{7} \stackrel{?}{=} \frac{19}{14}$$

$$\frac{(7/2) + 6}{14} \stackrel{?}{=} \frac{19}{14}$$

$$\frac{19}{28} \neq \frac{19}{14}$$

$x = \frac{1}{2}$ is not a solution.

(b) $x = 1$

$$\frac{1}{2} + \frac{6(1)}{7} \stackrel{?}{=} \frac{19}{14}$$

$$\frac{19}{14} = \frac{19}{14}$$

$x = 1$ is a solution.

(d) $x = 7$

$$\frac{7}{2} + \frac{6(7)}{7} = \frac{19}{14}$$

$$\frac{7}{2} + 6 = \frac{19}{14}$$

$$\frac{19}{2} \stackrel{?}{=} \frac{19}{14}$$

$x = 7$ is not a solution.

3. $3 + \frac{1}{x+2} = 4$

(a) $3 + \frac{1}{(-1) + 2} \stackrel{?}{=} 4$

$$4 = 4$$

$x = -1$ *is* a solution.

(c) $3 + \frac{1}{0 + 2} \stackrel{?}{=} 4$

$$\frac{7}{2} \neq 4$$

$x = 0$ *is not* a solution.

(b) $3 + \frac{1}{(-2) + 2} = 3 + \frac{1}{0}$ is undefined.

$x = -2$ *is not* a solution.

(d) $3 + \frac{1}{5 + 2} \stackrel{?}{=} 4$

$$\frac{22}{7} = 4$$

$x = 5$ *is not* a solution.

4. $\dfrac{(x+5)(x-3)}{2} = 24$

(a) $x = -3$

$$\frac{(-3+5)(-3-3)}{2} \stackrel{?}{=} 24$$

$$\frac{-12}{2} \stackrel{?}{=} 24$$

$$-6 \neq 24$$

$x = -3$ is not a solution.

(b) $x = -2$

$$\frac{(-2+5)(-2-3)}{2} \stackrel{?}{=} 24$$

$$\frac{-15}{2} \neq 24$$

$x = -2$ is not a solution.

(c) $x = 7$

$$\frac{(7+5)(7-3)}{2} \stackrel{?}{=} 24$$

$$24 = 24$$

$x = 7$ is a solution.

(d) $x = 9$

$$\frac{(9+5)(9-3)}{2} \stackrel{?}{=} 24$$

$$42 \neq 24$$

$x = 9$ is not a solution.

5. $\dfrac{\sqrt{x+4}}{6} + 3 = 4$

(a) $\dfrac{\sqrt{-3+4}}{6} + 3 \stackrel{?}{=} 4$

$$\frac{19}{6} \neq 4$$

$x = -3$ *is not* a solution.

(b) $\dfrac{\sqrt{0+4}}{6} + 3 \stackrel{?}{=} 4$

$$\frac{10}{3} \neq 4$$

$x = 0$ *is not* a solution.

(c) $\dfrac{\sqrt{21+4}}{6} + 3 \stackrel{?}{=} 4$

$$\frac{23}{6} \neq 4$$

$x = 21$ *is not* a solution.

(d) $\dfrac{\sqrt{32+4}}{6} + 3 \stackrel{?}{=} 4$

$$4 = 4$$

$x = 32$ *is* a solution.

6. $\dfrac{\sqrt[3]{x-8}}{3} = -\dfrac{2}{3}$

(a) $x = -16$

$$\frac{\sqrt[3]{-16-8}}{3} \stackrel{?}{=} -\frac{2}{3}$$

$$\sqrt[3]{-24} \neq -2$$

$x = -16$ is not a solution.

(b) $x = 0$

$$\frac{\sqrt[3]{0-8}}{3} \stackrel{?}{=} -\frac{2}{3}$$

$$-2 = -2$$

$x = 0$ is a solution.

(c) $x = 9$

$$\frac{\sqrt[3]{9-8}}{3} \stackrel{?}{=} -\frac{2}{3}$$

$$1 \neq -2$$

$x = 9$ is not a solution.

(d) $x = 16$

$$\frac{\sqrt[3]{16-8}}{3} \stackrel{?}{=} -\frac{2}{3}$$

$$2 \neq -2$$

$x = 16$ is not a solution.

7. $2(x - 1) = 2x - 2$ is an *identity* by the Distributive Property. It is true for all real values of x.

8. $-7(x - 3) + 4x = 3(7 - x)$ is an *identity* by simplification. It is true for all real values of x.

$$\begin{aligned} -7(x-3) + 4x &= -7x + 21 + 4x \\ &= 21 - 3x \\ &= 3(7 - x) \end{aligned}$$

9. $x^2 - 8x + 5 = (x - 4)^2 - 11$ is an *identity* since

$$\begin{aligned} (x-4)^2 - 11 &= x^2 - 8x + 16 - 11 \\ &= x^2 - 8x + 5. \end{aligned}$$

10. $x^2 + 2(3x - 2) = x^2 + 6x - 4$ is an *identity* by simplification. It is true for all real values of x.

11. $3 + \dfrac{1}{x+1} = \dfrac{4x}{x+1}$ is *conditional*. There are real values of x for which the equation is not true.

12. $\dfrac{5}{x} + \dfrac{3}{x} = 24$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 1$).

13. *Method 1:*

$$\begin{aligned} \frac{3x}{8} - \frac{4x}{3} &= 4 \\ \frac{9x - 32x}{24} &= 4 \\ -23x &= 96 \\ x &= -\frac{96}{23} \end{aligned}$$

Method 2: Graph $y_1 = \dfrac{3x}{8} - \dfrac{4x}{3}$ and $y_2 = 4$ in the same viewing window. These lines intersect at

$$x \approx -4.1739 \approx -\frac{96}{23}.$$

14. *Method 1:*

$$\begin{aligned} \frac{3z}{8} - \frac{z}{10} &= 6 \\ z\left(\frac{3}{8} - \frac{1}{10}\right) &= 6 \\ z\left(\frac{22}{80}\right) &= 6 \\ z = \frac{6(80)}{22} = \frac{240}{11} &\approx 21.8182 \end{aligned}$$

Method 2: Graph $y_1 = \dfrac{3x}{8} - \dfrac{x}{10}$ and $y_2 = 6$ in the same viewing window. These lines intersect at

$$x \approx 21.8182 \approx \frac{240}{11}.$$

15. *Method 1:*

$$\begin{aligned} \frac{2x}{5} + 5x &= \frac{4}{3} \\ \frac{2x + 25x}{5} &= \frac{4}{3} \\ 27x &= \frac{20}{3} \\ x = \frac{20}{3(27)} &= \frac{20}{81} \end{aligned}$$

Method 2: Graph $y_1 = \dfrac{2x}{5} + 5x$ and $y_2 = \dfrac{4}{3}$ in the same viewing window. These lines intersect at $x \approx 0.2469 \approx \dfrac{20}{81}$.

16. *Method 1:*

$$\begin{aligned} \frac{4y}{3} - 2y &= \frac{16}{5} \\ \frac{4y - 6y}{3} &= \frac{16}{5} \\ -2y &= \frac{48}{5} \\ y &= \frac{-24}{5} \end{aligned}$$

Method 2: Graph $y_1 = \dfrac{4x}{3} - 2x$ and $y_2 = \dfrac{16}{5}$ in the same viewing window. These lines intersect at $x = -4.8 = \dfrac{-24}{5}$.

17. $3x - 5 = 2x + 7$

$3x - 2x = 7 + 5$

$x = 12$

18. $5x + 3 = 6 - 2x$

$5x + 2x = 6 - 3$

$7x = 3$

$x = \frac{3}{7}$

19. $4y + 2 - 5y = 7 - 6y$

$-y + 2 = 7 - 6y$

$6y - y = 7 - 2$

$5y = 5$

$y = 1$

20. $5y + 1 = 8y - 5 + 6y$

$5y + 1 = 14y - 5$

$6 = 9y$

$y = \frac{2}{3}$

21. $3(y - 5) = 3 + 5y$

$3y - 15 = 3 + 5y$

$-18 = 2y$

$y = -9$

22. $5(z - 4) + 4z = 5 - 6z$

$5z - 20 + 4z = 5 - 6z$

$9z - 20 = 5 - 6z$

$15z = 25$

$z = \frac{25}{15} = \frac{5}{3}$

23. $\frac{x}{5} - \frac{x}{2} = 3$

$\frac{2x - 5x}{10} = 3$

$-3x = 30$

$x = -10$

24. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$

$\frac{5x}{4} - x = -\frac{1}{2} - \frac{1}{2}$

$\frac{1}{4}x = -1$

$x = -4$

25. $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$

$4\left(\frac{3}{2}\right)(z + 5) - 4\left(\frac{1}{4}\right)(z + 24) = 4(0)$

$6(z + 5) - (z + 24) = 0$

$6z + 30 - z - 24 = 0$

$5z = -6$

$z = -\frac{6}{5}$

26. $\frac{3x}{2} + \frac{1}{4}(x - 2) = 10$

$(4)\left(\frac{3x}{2}\right) + (4)\frac{1}{4}(x - 2) = (4)10$

$6x + (x - 2) = 40$

$7x - 2 = 40$

$7x = 42$

$x = 6$

27. $\frac{2(z - 4)}{5} + 5 = 10z$

$\frac{(2z - 8) + 25}{5} = 10z$

$2z + 17 = 50z$

$17 = 48z$

$z = \frac{17}{48}$

28. $\frac{5}{3} + 2(y + 1) = \frac{10}{3}$

$5 + 6(y + 1) = 10$

$6y + 6 = 5$

$6y = -1$

$y = -\frac{1}{6}$

29.
$$\frac{100-4u}{3} = \frac{5u+6}{4} + 6$$
$$12\left(\frac{100-4u}{3}\right) = 12\left(\frac{5u+6}{4}\right) + 12(6)$$
$$4(100-4u) = 3(5u+6) + 72$$
$$400 - 16u = 15u + 18 + 72$$
$$-31u = -310$$
$$u = 10$$

30.
$$\frac{17+y}{y} + \frac{32+y}{y} = 100$$
$$(y)\frac{17+y}{y} + (y)\frac{32+y}{y} = 100(y)$$
$$17 + y + 32 + y = 100y$$
$$49 + 2y = 100y$$
$$49 = 98y$$
$$\frac{1}{2} = y$$

31.
$$\frac{5x-4}{5x+4} = \frac{2}{3}$$
$$3(5x-4) = 2(5x+4)$$
$$15x - 12 = 10x + 8$$
$$5x = 20$$
$$x = 4$$

32.
$$\frac{10x+3}{5x+6} = \frac{1}{2}$$
$$20x + 6 = 5x + 6$$
$$15x = 0$$
$$x = 0$$

33.
$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$$
$$\frac{(x+3)+(x-3)}{x^2-9} = \frac{10}{x^2-9}$$
$$2x = 10$$
$$x = 5$$

34.
$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6}$$
$$(x^2+x-6)\frac{1}{x-2} + (x^2+x-6)\frac{3}{x+3} = (x^2+x-6)\frac{4}{x^2+x-6}$$
$$(x+3) + 3(x-2) = 4$$
$$x + 3 + 3x - 6 = 4$$
$$4x - 3 = 4$$
$$4x = 7$$
$$x = \frac{7}{4}$$

35.
$$\frac{7}{2x+1} - \frac{8x}{2x-1} = -4$$
$$7(2x-1) - 8x(2x+1) = -4(2x+1)(2x-1)$$
$$14x - 7 - 16x^2 - 8x = -16x^2 + 4$$
$$6x = 11$$
$$x = \frac{11}{6}$$

36.
$$\frac{x}{x+4} + \frac{4}{x+4} + 2 = 0$$
$$\frac{x+4}{x+4} + 2 = 0$$
$$1 + 2 = 0$$

Impossible

No solution

37. $\frac{1}{x} + \frac{2}{x-5} = 0$

$1(x-5) + 2x = 0$

$3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$

38. $3 = 2 + \frac{2}{z+2}$

$1 = \frac{2}{z+2}$

$z + 2 = 2$

$z = 0$

39. $\frac{3}{x(x-3)} + \frac{4}{x} = \frac{1}{x-3}$

$3 + 4(x-3) = x$

$3 + 4x - 12 = x$

$3x = 9$

$x = 3$

A check reveals that $x = 3$ is an extraneous solution, so there is no solution.

40. $\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)}$

$x(x+3)\frac{6}{x} - x(x+3)\frac{2}{x+3} = x(x+3)\frac{3(x+5)}{x(x+3)}$

$6(x+3) - 2x = 3(x+5)$

$6x + 18 - 2x = 3x + 15$

$4x + 18 = 3x + 15$

$x = -3$

Check: $\frac{6}{-3} - \frac{2}{-3+3} = \frac{3(-3+5)}{-3(-3+3)}$

$-2 - \frac{2}{0} = \frac{6}{-3(0)}$

Division by zero is undefined. Thus, $x = -3$ is not a solution, and the original equation has no solution.

41. $y = x - 5$

Let $y = 0$: $0 = x - 5 \implies x = 5 \implies (5, 0)$ x-intercept

Let $x = 0$: $y = 0 - 5 \implies y = -5 \implies (0, -5)$ y-intercept

42. $y = -\frac{3}{4}x - 3$

Let $y = 0$: $0 = -\frac{3}{4}x - 3 \implies \frac{3}{4}x = -3 \implies x = -4 \implies (-4, 0)$ x-intercept

Let $x = 0$: $y = -\frac{3}{4}(0) - 3 = -3 \implies (0, -3)$ y-intercept

43. $y = x^2 + x - 2$

Let $y = 0$: $(x^2 + x - 2) = (x+2)(x-1) = 0 \implies x = -2, 1 \implies (-2, 0), (1, 0)$ x-intercepts

Let $x = 0$: $y = 0^2 + 0 - 2 = -2 \implies (0, -2)$ y-intercept

44. $y = 4 - x^2$

Let $y = 0$: $0 = 4 - x^2 \implies x = 2, -2 \implies (2, 0), (-2, 0)$ x-intercepts

Let $x = 0$: $y = 4 - 0^2 = 4 \implies (0, 4)$ y-intercept

45. $y = x\sqrt{x+2}$

Let $y = 0$: $0 = x\sqrt{x+2} \implies x = 0, -2 \implies (0, 0), (-2, 0)$ x-intercepts

Let $x = 0$: $y = 0\sqrt{0+2} = 0 \implies (0, 0)$ y-intercept

46. $y = -\frac{1}{2}x\sqrt{x+3} + 1$

Let $y = 0$: $0 = -\frac{1}{2}x\sqrt{x+3} + 1 \implies \frac{1}{2}x\sqrt{x+3} = 1 \implies x\sqrt{x+3} = 2$

$\implies x^2(x+3) = 4 \implies x^3 + 3x^2 - 4 = 0$

$\implies (x-1)(x^2+4x+4) = 0 \implies (x-1)(x+2)^2 = 0$

$\implies x = 1 \implies (1, 0)$ $(x = -2$ is impossible.)

Let $x = 0 \implies y = 1 \implies (0, 1)$ y-intercept

47. $xy = 4$

If $x = 0$, then $0y = 0 = 4$, which is impossible. Similarly, $y = 0$ is impossible.
Hence there are no intercepts.

48. $4xy = 3x - 1$

Let $y = 0$: $0 = 3x - 1 \implies x = \frac{1}{3} \implies \left(\frac{1}{3}, 0\right)$ x-intercept

Let $x = 0$: $0 = -1$ is impossible. No y-intercepts

49. $y = |x-2| - 4$

Let $y = 0$: $|x-2| - 4 = 0 \implies |x-2| = 4 \implies x = -2, 6 \implies (-2, 0), (6, 0)$ x-intercepts

Let $x = 0$: $|0-2| - 4 = |-2| - 4 = 2 - 4 = -2 = y \implies (0, -2)$ y-intercept

50. $y = 3 - \frac{1}{2}|x+1|$

Let $y = 0$: $0 = 3 - \frac{1}{2}|x+1| \implies \frac{1}{2}|x+1| = 3 \implies |x+1| = 6$

$\implies x = 5, -7 \implies (5, 0), (-7, 0)$ x-intercepts

Let $x = 0$: $y = 3 - \frac{1}{2} = 2.5, \left(0, \frac{5}{2}\right)$ y-intercept

51. $xy - 2y - x + 1 = 0$

Let $y = 0$:

$-x + 1 = 0 \implies x = 1 \implies (1, 0)$ x-intercept

Let $x = 0$:

$-2y + 1 = 0 \implies y = \frac{1}{2} \implies \left(0, \frac{1}{2}\right)$ y-intercept

52. $xy - x + 4y = 0$

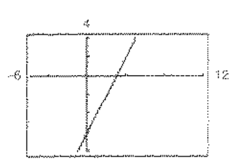
Let $y = 0$:

$-x = 0 \implies x = 0 \implies (0, 0)$ x-intercept

Let $x = 0$:

$4y = 0 \implies y = 0 \implies (0, 0)$ y-intercept

53.



$y = 0 = 2(x-1) - 4$

$= 2x - 2 - 4$

$= 2x - 6 \implies 2x = 6 \implies x = 3$

$(3, 0)$

54. $y = 4(x+3) - 2$

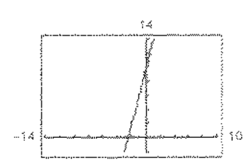
$0 = 4(x+3) - 2$

$0 = 4x + 10$

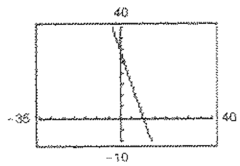
$4x = -10$

$x = -\frac{5}{2}$

$\left(-\frac{5}{2}, 0\right)$



55.

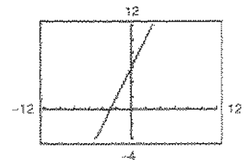


$$y = 0 = 20 - (3x - 10)$$
$$= 20 - 3x + 10$$
$$= 30 - 3x \implies 3x = 30 \implies x = 10$$

$(10, 0)$

56.

$$0 = 10 + 2(x - 2)$$
$$0 = 10 + 2x - 4$$
$$0 = 6 + 2x$$
$$-2x = 6$$
$$x = -3$$

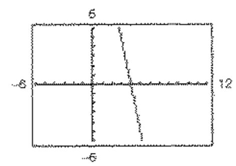


Intercept: $(-3, 0)$

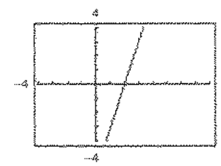
The solution to $0 = 10 + 2(x - 2)$ is the same as the x-intercept of $0 = 10 + 2(x - 2)$. They are both $x = -3$.

57.

$$f(x) = 5(4 - x)$$
$$5(4 - x) = 0$$
$$4 - x = 0$$
$$x = 4$$

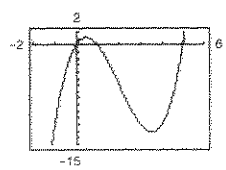


58. $f(x) = 3(x - 5) + 9$

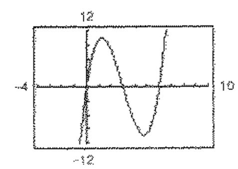


59.

$$f(x) = x^3 - 6x^2 + 5x$$
$$x^3 - 6x^2 + 5x = 0$$
$$x(x^2 - 6x + 5) = 0$$
$$x(x - 5)(x - 1) = 0$$
$$x = 0, 5, 1$$

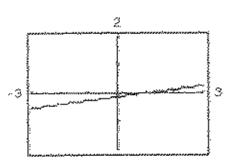


60. $f(x) = x^3 - 9x^2 + 18x$

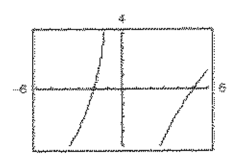


61.

$$f(x) = \frac{x + 2}{3} - \frac{x - 1}{5} - 1$$
$$\frac{x + 2}{3} - \frac{x - 1}{5} - 1 = 0$$
$$5(x + 2) - 3(x - 1) - 15 = 0$$
$$2x = 2$$
$$x = 1$$

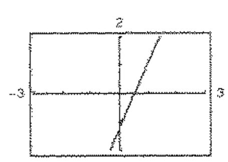


62. $f(x) = x - 3 - \dfrac{10}{x}$



63. $2.7x - 0.4x = 1.2$

$$2.3x = 1.2$$
$$x = \frac{1.2}{2.3} \approx 0.522$$



$$f(x) = 2.7x - 0.4x - 1.2 = 0$$
$$x \approx 0.522$$

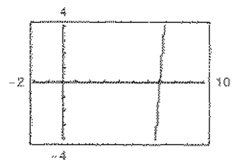
64. $3.5x - 8 = 0.5x$

$$3x = 8$$
$$x = \frac{8}{3}$$
$$f(x) = 3.5x - 8 - 0.5x = 0$$
$$x = 2.667$$

65. $$\begin{aligned} 25(x-3) &= 12(x+2)-10 \\ 25x-75 &= 12x+24-10 \\ 13x-89 &= 0 \\ x &= \tfrac{89}{13} \end{aligned}$$

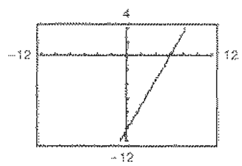
$f(x) = 25(x-3) - 12(x+2) + 10 = 0$

$x = 6.846$



66. $$\begin{aligned} 1200 &= 300 + 2(x-500) \\ 900 &= 2x - 1000 \\ 1900 &= 2x \\ x &= 950 \end{aligned}$$

67. $$\begin{aligned} \frac{3x}{2} + \frac{1}{4}(x-2) &= 10 \\ \frac{6x}{4} + \frac{x}{4} &= 10 + \frac{1}{2} \\ \frac{7x}{4} &= \frac{21}{2} \\ x &= 6 \end{aligned}$$

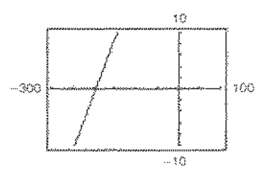


$f(x) = \dfrac{3x}{2} + \dfrac{1}{4}(x-2) - 10 = 0$

$x = 6.0$

68. $$\begin{aligned} \frac{2x}{3} + \frac{1}{2}(x-5) &= 6 \\ \left(\frac{2}{3} + \frac{1}{2}\right)x &= \frac{5}{2} + 6 \\ \frac{7}{6}x &= \frac{17}{2} \\ x &= \frac{51}{7} \approx 7.286 \end{aligned}$$

69. $$\begin{aligned} 0.60x + 0.40(100-x) &= 1.2 \\ 0.60x + 40 - 0.40x &= 1.2 \\ 0.20x &= -38.8 \\ x &= -194 \end{aligned}$$

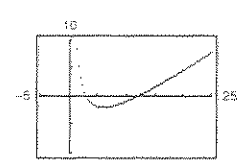


$f(x) = 0.60x + 0.40(100-x) - 1.2 = 0$

$x = -194$

70. $$\begin{aligned} 0.75x + 0.2(80-x) &= 20 \\ (0.75-0.2)x &= 20-16 \\ 0.55x &= 4 \\ x &= \frac{4}{0.55} = \frac{80}{11} \approx 7.273 \end{aligned}$$

71. $$\begin{aligned} \frac{2x}{3} &= 10 - \frac{24}{x} \\ \frac{2x}{3}(3x) &= 10(3x) - \frac{24}{x}(3x) \\ 2x^2 &= 30x - 72 \\ 2x^2 - 30x + 72 &= 0 \\ x^2 - 15x + 36 &= 0 \\ (x-3)(x-12) &= 0 \\ x &= 3, 12 \end{aligned}$$



$f(x) = \dfrac{2x}{3} - 10 + \dfrac{24}{x}$

$x = 3, 12$

72. $\dfrac{x-3}{25} = \dfrac{x-5}{12}$

$12(x-3) = 25(x-5)$

$12x - 36 = 25x - 125$

$13x = 89$

$x = \dfrac{89}{13} \approx 6.846$

73. $\dfrac{3}{x+2} - \dfrac{4}{x-2} = 5$

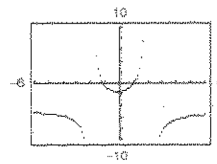
$3(x-2) - 4(x+2) = 5(x+2)(x-2)$

$3x - 6 - 4x - 8 = 5(x^2 - 4)$

$0 = 5x^2 + x - 6$

$0 = (x-1)(5x+6)$

$x = 1, -\dfrac{6}{5}$



$f(x) = \dfrac{3}{x+2} - \dfrac{4}{x-2} - 5$

$= 0$

$x = 1.0, -1.2$

74. $\dfrac{6}{x} + \dfrac{8}{x+5} = 3$

$6(x+5) + 8x = 3x(x+5)$

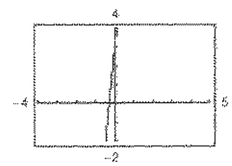
$6x + 30 + 8x = 3x^2 + 15x$

$0 = 3x^2 + x - 30$

$= (x-3)(3x+10)$

$x = 3, -\dfrac{10}{3}$

75. $(x+2)^2 = x^2 - 6x + 1$



$x^2 + 4x + 4 = x^2 - 6x + 1$

$10x = -3$

$x = -\dfrac{3}{10}$

$f(x) = (x+2)^2 - x^2 + 6x - 1$

$x = -\dfrac{3}{10}$

76. $(x+1)^2 + 2(x-2) = (x+1)(x-2)$

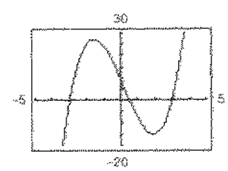
$x^2 + 2x + 1 + 2x - 4 = x^2 - x - 2$

$5x = 1$

$x = \frac{1}{5}$

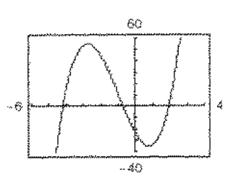
77. $2x^3 - x^2 - 18x + 9 = 0$

$x = -3.0, 0.5, 3.0$



78. $4x^3 + 12x^2 - 26x - 24 = 0$

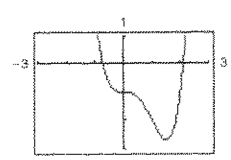
$x = -4.206, -0.735, 1.941$



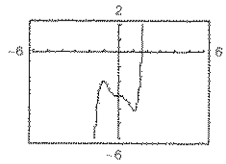
79. $x^4 = 2x^3 + 1$

$x^4 - 2x^3 - 1 = 0$

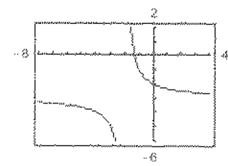
$x \approx -0.717, 2.107$



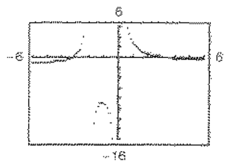
80. $$\begin{aligned} x^5 &= 3 + 2x^3 \\ x^5 - 3 - 2x^3 &= 0 \\ x &= 1.638 \end{aligned}$$



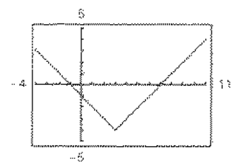
81. $$\begin{aligned} \frac{2}{x+2} &= 3 \\ \frac{2}{x+2} - 3 &= 0 \\ x &= -\frac{4}{3} \end{aligned}$$



82. $$\begin{aligned} \frac{5}{x} &= 1 + \frac{3}{x+2} \\ \frac{5}{x} - 1 - \frac{3}{x+2} &= 0 \\ x &= -3.162, 3.162 \end{aligned}$$

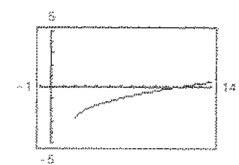


83. $$\begin{aligned} |x - 3| &= 4 \\ |x - 3| - 4 &= 0 \\ x &= -1, 7 \end{aligned}$$



84. $$\begin{aligned} |x + 1| &= 6 \\ x + 1 = 6 \quad &\text{or} \quad x + 1 = -6 \\ x = 5 \quad &\text{or} \quad x = -7 \end{aligned}$$

85. $$\begin{aligned} \sqrt{x-2} &= 3 \\ \sqrt{x-2} - 3 &= 0 \\ x &= 11 \end{aligned}$$



86. $$\begin{aligned} \sqrt{x-4} &= 8 \\ x - 4 &= 64 \\ x &= 68 \end{aligned}$$

87. $$\begin{aligned} y &= 2 - x \\ y &= 2x - 1 \\ 2 - x &= 2x - 1 \\ 3 &= 3x \\ x &= 1, y = 2 - 1 = 1 \\ (x, y) &= (1, 1) \end{aligned}$$

88. $$\begin{aligned} y &= x + 4 \\ y &= \tfrac{5}{2} - \tfrac{1}{2}x \\ x + 4 &= \tfrac{5}{2} - \tfrac{1}{2}x \\ 2x + 8 &= 5 - x \\ 3x &= -3 \\ x &= -1 \\ y &= -1 + 4 = 3 \end{aligned}$$

Solution: $(-1, 3)$

89. $$\begin{aligned} x - y = -4 &\implies y = x + 4 \\ x^2 - y = -2 &\implies y = x^2 + 2 \\ x^2 + 2 &= x + 4 \\ x^2 - x - 2 &= 0 \\ (x - 2)(x + 1) &= 0 \\ x &= 2, y = 6 \\ x &= -1, y = 3 \end{aligned}$$

$(2, 6), (-1, 3)$

90. $$\begin{aligned} 3x + y = 2 &\implies y = 2 - 3x \\ x^3 + y = 0 &\implies y = -x^3 \\ 2 - 3x &= -x^3 \\ x^3 - 3x + 2 &= 0 \\ (x - 1)(x^2 + x - 2) &= 0 \\ (x - 1)(x + 2)(x - 1) &= 0 \\ x &= 1, -2 \end{aligned}$$

$(x, y) = (1, -1), (-2, 8)$

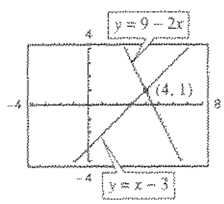
91.
$$y = x^2 - x + 1$$
$$y = x^2 + 2x + 4$$
$$x^2 - x + 1 = x^2 + 2x + 4$$
$$-3 = 3x$$
$$x = -1$$
$$y = (-1)^2 - (-1) + 1 = 3$$
$$(x, y) = (-1, 3)$$

92.
$$y = -x^2 + 3x + 1$$
$$y = -x^2 - 2x - 4$$
$$-x^2 + 3x + 1 = -x^2 - 2x - 4$$
$$3x + 1 = -2x - 4$$
$$5x = -5$$
$$x = -1$$
$$y = -(-1)^2 + 3(-1) + 1 = -3$$
$$(x, y) = (-1, -3)$$

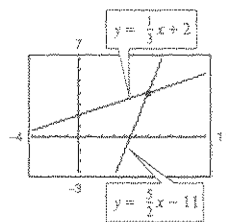
93. $y = 9 - 2x$

$y = x - 3$

$(4, 1)$



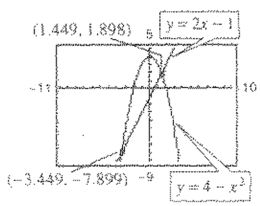
94. $(x, y) = (6, 4)$



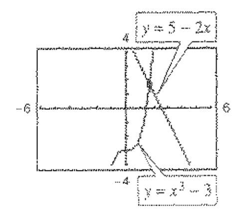
95. $y = 4 - x^2$

$y = 2x - 1$

$(x, y) = (1.449, 1.898), (-3.449, -7.899)$



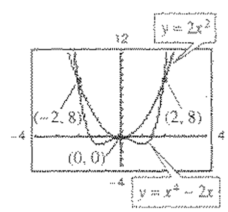
96. $(x, y) = (1.670, 1.660)$



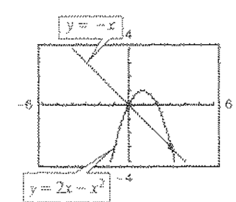
97. $y = 2x^2$

$y = x^4 - 2x^2$

$(x, y) = (0, 0), (2, 8), (-2, 8)$



98. $(x, y) = (0, 0), (3, -3)$



99.
$$6x^2 + 3x = 0$$
$$3x(2x + 1) = 0$$
$$3x = 0 \text{ or } 2x + 1 = 0$$
$$x = 0 \text{ or } \quad x = -\tfrac{1}{2}$$

100.
$$9x^2 - 1 = 0$$
$$(3x + 1)(3x - 1) = 0$$
$$3x + 1 = 0 \implies x = -\tfrac{1}{3}$$
$$3x - 1 = 0 \implies x = \tfrac{1}{3}$$

101.
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \text{ or } x + 2 = 0$$
$$x = 4 \text{ or } \quad x = -2$$

102.
$$x^2 - 10x + 9 = 0$$
$$(x - 9)(x - 1) = 0$$
$$x - 9 = 0 \implies x = 9$$
$$x - 1 = 0 \implies x = 1$$

103.
$$3 + 5x - 2x^2 = 0$$
$$(3 - x)(1 + 2x) = 0$$
$$3 - x = 0 \text{ or } 1 + 2x = 0$$
$$x = 3 \text{ or } \quad x = -\tfrac{1}{2}$$

104.
$$2x^2 = 19x + 33$$
$$2x^2 - 19x - 33 = 0$$
$$(2x + 3)(x - 11) = 0$$
$$2x + 3 = 0 \implies x = -\tfrac{3}{2}$$
$$x - 11 = 0 \implies x = 11$$

105. $x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ or $x = 2$

106. $-x^2 + 8x = 12$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2, 6$

107. $(x + a)^2 - b^2 = 0$

$[(x + a) + b][(x + a) - b] = 0$

$x + a + b = 0 \Longrightarrow x = -a - b$

$x + a - b = 0 \Longrightarrow x = -a + b$

108. $x^2 + 2ax + a^2 = 0$

$(x + a)^2 = 0$

$x = -a$

109. $x^2 = 49$

$x = \pm\sqrt{49} = \pm 7$

110. $x^2 = 144$

$x = \pm\sqrt{144} = \pm 12$

111. $(x - 12)^2 = 16$

$x - 12 = \pm\sqrt{16} = \pm 4$

$x = 12 \pm 4$

$x = 16, 8$

112. $(x - 5)^2 = 25$

$x - 5 = \pm 5$

$x = 5 \pm 5$

$x = 10, 0$

113. $(3x - 1)^2 + 6 = 0$

$(3x - 1)^2 = -6$

$3x - 1 = \pm\sqrt{-6} = \pm\sqrt{6}i$

$x = \dfrac{1}{3} \pm \dfrac{\sqrt{6}}{3}i \approx 0.33 \pm 0.82i$

114. $(2x + 3)^2 + 25 = 0$

$(2x + 3)^2 = -25$

$2x + 3 = \pm\sqrt{-25} = \pm 5i$

$x = -\dfrac{3}{2} \pm \dfrac{5}{2}i = -1.5 \pm 2.5i$

115. $(2x - 1)^2 = 12$

$2x - 1 = \pm\sqrt{12} = \pm 2\sqrt{3}$

$2x = 1 \pm 2\sqrt{3}$

$x = \frac{1}{2} \pm \sqrt{3}$

$x = 2.23, -1.23$

116. $(4x + 7)^2 = 44$

$4x + 7 = \pm\sqrt{44} = \pm 2\sqrt{11}$

$4x = -7 \pm 2\sqrt{11}$

$x = -\frac{7}{4} \pm \frac{1}{2}\sqrt{11}$

$x \approx -3.41, -0.09$

117. $(x - 7)^2 = (x + 3)^2$

$x - 7 = \pm(x + 3)$

$x - 7 = x + 3,$ impossible

$x - 7 = -(x + 3) \Longrightarrow 2x = 4$

$\Longrightarrow x = 2$

118. $(x + 5)^2 = (x + 4)^2$

$x + 5 = x + 4,$ impossible

$x + 5 = -(x + 4)$

$2x = -9$

$x = -\frac{9}{2} = -4.5$

119. $x^2 + 4x = 32$

$x^2 + 4x + 4 = 32 + 4$

$(x + 2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = -8, 4$

120. $x^2 - 2x - 3 = 0$

$x^2 - 2x + 1 = 3 + 1$

$(x - 1)^2 = 4$

$x - 1 = \pm 2$

$x = 1 \pm 2$

$x = 3, -1$

121. $x^2 + 6x + 2 = 0$

$x^2 + 6x = -2$

$x^2 + 6x + 3^2 = -2 + 3^2$

$(x + 3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

122. $x^2 + 8x + 14 = 0$

$$x^2 + 8x = -14$$
$$x^2 + 8x + 4^2 = -14 + 16$$
$$(x + 4)^2 = 2$$
$$x + 4 = \pm\sqrt{2}$$
$$x = -4 \pm \sqrt{2}$$

123. $9x^2 - 18x + 3 = 0$

$$x^2 - 2x + \frac{1}{3} = 0$$
$$x^2 - 2x = -\frac{1}{3}$$
$$x^2 - 2x + 1^2 = -\frac{1}{3} + 1^2$$
$$(x - 1)^2 = \frac{2}{3}$$
$$x - 1 = \pm\sqrt{\frac{2}{3}}$$
$$x = 1 \pm \sqrt{\frac{2}{3}}$$
$$x = 1 \pm \frac{\sqrt{6}}{3}$$

124. $4x^2 - 4x - 99 = 0$

$$4\left(x^2 - x + \frac{1}{4}\right) = 99 + 1$$
$$4\left(x - \frac{1}{2}\right)^2 = 100$$
$$\left(x - \frac{1}{2}\right)^2 = 25$$
$$x - \frac{1}{2} = \pm 5$$
$$x = \frac{1}{2} \pm 5$$
$$x = -\frac{9}{2}, \frac{11}{2}$$

125. $-6 + 2x - x^2 = 0$

$$(x^2 - 2x + 1) = -6 + 1$$
$$(x - 1)^2 = -5$$
$$x - 1 = \pm\sqrt{-5}$$
$$= \pm\sqrt{5}i$$
$$x = 1 \pm \sqrt{5}i$$

126. $-x^2 + x - 1 = 0$

$$x^2 - x + \frac{1}{4} = -1 + \frac{1}{4}$$
$$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$$
$$x - \frac{1}{2} = \pm\frac{\sqrt{3}}{2}i$$
$$x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

127. $2x^2 + 5x - 8 = 0$

$$x^2 + \frac{5}{2}x - 4 = 0$$
$$x^2 + \frac{5}{2}x + \frac{25}{16} = 4 + \frac{25}{16}$$
$$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$$
$$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$$
$$x = \frac{-5}{4} \pm \frac{\sqrt{89}}{4}$$

128. $9x^2 - 12x = 14$

$$x^2 - \frac{4}{3}x = \frac{14}{9}$$
$$x^2 - \frac{4}{3}x + \left(\frac{2}{3}\right)^2 = \frac{14}{9} + \frac{4}{9}$$
$$\left(x - \frac{2}{3}\right)^2 = \frac{18}{9}$$
$$\left(x - \frac{2}{3}\right)^2 = 2$$
$$x - \frac{2}{3} = \pm\sqrt{2}$$
$$x = \frac{2}{3} \pm \sqrt{2}$$

129. $-x^2 + 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)}$$
$$= \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3}$$

130. $x^2 - 10x + 22 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 88}}{2}$$

$$= \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3}$$

131. $x^2 + 8x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5}$$

132. $4x^2 - 4x - 4 = 0$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

133. $x^2 + 3x + 8 = 0$

$$x = \frac{-3 \pm \sqrt{9 - 4(8)}}{2}$$

$$= \frac{-3 \pm \sqrt{-23}}{2}$$

$$= -\frac{3}{2} \pm \frac{\sqrt{23}i}{2}$$

134. $x^2 + 5x + 16 = 0$

$$x = \frac{-5 \pm \sqrt{25 - 4(16)}}{2}$$

$$= \frac{-5 \pm \sqrt{-39}}{2}$$

$$= -\frac{5}{2} \pm \frac{\sqrt{39}}{2}i$$

135. $28x - 49x^2 = 4$

$$-49x^2 + 28x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$

$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

136. $9x^2 + 24x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-24 \pm \sqrt{24^2 - 4(9)(16)}}{2(9)}$$

$$= \frac{-24 \pm \sqrt{576 - 576}}{18}$$

$$= -\frac{4}{3}$$

137. $4x^2 + 16x + 17 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-16 \pm \sqrt{16^2 - 4(4)(17)}}{2(4)}$$

$$= \frac{-16 \pm \sqrt{-16}}{8}$$

$$= \frac{-16 \pm 4i}{8}$$

$$= -2 \pm \frac{1}{2}i$$

138. $9x^2 - 6x + 37 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{6 \pm \sqrt{36 - 1332}}{18}$$
$$= \frac{1}{3} \pm 2i$$

139. $x^2 - 2x - 1 = 0$

$$x^2 - 2x = 1$$
$$x^2 - 2x + 1^2 = 1 + 1^2$$
$$(x - 1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

140. $11x^2 + 33x = 0$

$$11(x^2 + 3x) = 0$$
$$x(x + 3) = 0$$
$$x = 0$$
$$x + 3 = 0 \implies x = -3$$

141. $(x + 3)^2 = 81$

$$x + 3 = \pm 9$$
$$x + 3 = 9 \text{ or } x + 3 = -9$$
$$x = 6 \text{ or } \quad x = -12$$

142. $(x - 1)^2 = -1$

$$x - 1 = \pm\sqrt{-1} = \pm i$$
$$x = 1 \pm i$$

143. $x^2 - 14x + 49 = 0$

$$(x - 7)^2 = 0$$
$$x = 7$$

144. $x^2 - 2x + \frac{13}{4} = 0$

$$x = \frac{2 \pm \sqrt{4 - 4(13/4)}}{2}$$
$$= \frac{2 \pm \sqrt{-9}}{2}$$
$$= 1 \pm \frac{3}{2}i$$

145. $x^2 - x - \frac{11}{4} = 0$

$$x^2 - x + \frac{1}{4} = \frac{11}{4} + \frac{1}{4}$$
$$\left(x - \frac{1}{2}\right)^2 = 3$$
$$x - \frac{1}{2} = \pm\sqrt{3}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$

146. $x^2 + 3x - \frac{3}{4} = 0$

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$
$$x = -\frac{3}{2} \pm \sqrt{3}$$

147. $(x + 1)^2 = x^2$

$$(x + 1)^2 - x^2 = 0$$
$$(x + 1 - x)(x + 1 + x) = 0$$
$$2x + 1 = 0$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

148. $a^2x^2 - b^2 = 0, \ a \neq 0$

$$a^2x^2 = b^2$$
$$x^2 = \frac{b^2}{a^2}$$
$$x = \pm\frac{b}{a}$$

(If $b = 0$, then $x = 0$.)

149. $4x^4 - 16x^2 = 0$

$$4x^2(x^2 - 4) = 0$$
$$4x^2(x - 2)(x + 2) = 0$$
$$x = 0, \pm 2$$

150. $8x^4 - 18x^2 = 0$

$$2x^2(4x^2 - 9) = 0$$
$$2x^2(2x + 3)(2x - 3) = 0$$
$$x = 0, \pm\tfrac{3}{2}$$

151. $5x^3 + 30x^2 + 45x = 0$

$$5x(x^2 + 6x + 9) = 0$$
$$5x(x + 3)^2 = 0$$
$$5x = 0 \implies x = 0$$
$$x + 3 = 0 \implies x = -3$$

152. $9x^4 - 24x^3 + 16x^2 = 0$

$$x^2(9x^2 - 24x + 16) = 0$$
$$x^2(3x - 4)^2 = 0$$
$$x^2 = 0 \implies x = 0$$
$$3x - 4 = 0 \implies x = \tfrac{4}{3}$$

153. $4x^4 - 18x^2 = 0$

$2x^2(2x^2 - 9) = 0$

$2x^2 = 0 \implies x = 0$

$2x^2 - 9 = 0 \implies x = \pm\dfrac{3\sqrt{2}}{2}$

154. $20x^3 - 125x = 0$

$5x(4x^2 - 25) = 0$

$5x(2x + 5)(2x - 5) = 0$

$5x = 0 \implies x = 0$

$2x + 5 = 0 \implies x = -\dfrac{5}{2}$

$2x - 5 = 0 \implies x = \dfrac{5}{2}$

155. $x^4 - 4x^2 + 3 = 0$

$(x^2 - 3)(x^2 - 1) = 0$

$(x + \sqrt{3})(x - \sqrt{3})(x + 1)(x - 1) = 0$

$x + \sqrt{3} = 0 \implies x = -\sqrt{3}$

$x - \sqrt{3} = 0 \implies x = \sqrt{3}$

$x + 1 = 0 \implies x = -1$

$x - 1 = 0 \implies x = 1$

156. $x^4 + 5x^2 - 36 = 0$

$(x^2 + 9)(x^2 - 4) = 0$

$(x^2 + 9)(x + 2)(x - 2) = 0$

$x^2 + 9 = 0 \implies x = \pm 3i$

$x + 2 = 0 \implies x = -2$

$x - 2 = 0 \implies x = 2$

157. $x^3 - 3x^2 - x + 3 = 0$

$x^2(x - 3) - (x - 3) = 0$

$(x - 3)(x^2 - 1) = 0$

$(x - 3)(x + 1)(x - 1) = 0$

$x - 3 = 0 \implies x = 3$

$x + 1 = 0 \implies x = -1$

$x - 1 = 0 \implies x = 1$

158. $x^4 + 2x^3 - 8x - 16 = 0$

$x^3(x + 2) - 8(x + 2) = 0$

$(x^3 - 8)(x + 2) = 0$

$(x - 2)(x^2 + 2x + 4)(x + 2) = 0$

$x - 2 = 0 \implies x = 2$

$x^2 + 2x + 4 = 0 \implies x = -1 \pm \sqrt{3}i$

$x + 2 = 0 \implies x = -2$

159. $4x^4 - 65x^2 + 16 = 0$

$(4x^2 - 1)(x^2 - 16) = 0$

$(2x + 1)(2x - 1)(x + 4)(x - 4) = 0$

$2x + 1 = 0 \implies x = -\frac{1}{2}$

$2x - 1 = 0 \implies x = \frac{1}{2}$

$x + 4 = 0 \implies x = -4$

$x - 4 = 0 \implies x = 4$

160.
$$36t^4 + 29t^2 - 7 = 0$$
$$(36t^2 - 7)(t^2 + 1) = 0$$
$$(6t + \sqrt{7})(6t - \sqrt{7})(t^2 + 1) = 0$$
$$6t + \sqrt{7} = 0 \implies t = -\frac{\sqrt{7}}{6}$$
$$6t - \sqrt{7} = 0 \implies t = \frac{\sqrt{7}}{6}$$
$$t^2 + 1 = 0 \implies t = \pm i$$

161.
$$\frac{1}{t^2} + \frac{8}{t} + 15 = 0$$
$$1 + 8t + 15t^2 = 0$$
$$(1 + 3t)(1 + 5t) = 0$$
$$1 + 3t = 0 \implies t = -\frac{1}{3}$$
$$1 + 5t = 0 \implies t = -\frac{1}{5}$$

162.
$$6 - \frac{1}{x} - \frac{1}{x^2} = 0$$
$$6x^2 - x - 1 = 0$$
$$(3x + 1)(2x - 1) = 0$$
$$x = -\frac{1}{3}, \frac{1}{2}$$

163. $6\left(\frac{s}{s+1}\right)^2 + 5\left(\frac{s}{s+1}\right) - 6 = 0$

Let $u = \frac{s}{(s+1)}$.
$$6u^2 + 5u - 6 = 0$$
$$(3u - 2)(2u + 3) = 0$$
$$3u - 2 = 0 \implies u = \frac{2}{3}$$
$$2u + 3 = 0 \implies u = -\frac{3}{2}$$
$$\frac{s}{s+1} = \frac{2}{3} \implies s = 2$$
$$\frac{s}{s+1} = -\frac{3}{2} \implies s = -\frac{3}{5}$$

164. $8\left(\frac{t}{t-1}\right)^2 - 2\left(\frac{t}{t-1}\right) - 3 = 0$

Let $\frac{t}{t-1} = x$. Then:
$$8x^2 - 2x - 3 = 0$$
$$(2x + 1)(4x - 3) = 0$$
$$x = -\frac{1}{2}, x = \frac{3}{4}$$
$$\frac{t}{t-1} = -\frac{1}{2} \implies 2t = -t + 1 \implies t = \frac{1}{3}$$
$$\frac{t}{t-1} = \frac{3}{4} \implies 4t = 3t - 3 \implies t = -3$$

165.
$$2x + 9\sqrt{x} - 5 = 0$$
$$(2\sqrt{x} - 1)(\sqrt{x} + 5) = 0$$
$$\sqrt{x} = \tfrac{1}{2} \implies x = \tfrac{1}{4}$$
($\sqrt{x} = -5$ is not possible.)

Note: You can see graphically that there is only one solution.

166. $6x - 7\sqrt{x} - 3 = 0$

Let $z = \sqrt{x}$. Then:
$$6z^2 - 7z - 3 = 0$$
$$(2z - 3)(3z + 1) = 0$$
$$z = \tfrac{3}{2}, -\tfrac{1}{3}$$
Hence, $x = \frac{9}{4}$. (The other value is extraneous.)

167. $\sqrt{x-10}-4=0$

$\sqrt{x-10}=4$

$x-10=16$

$x=26$

168. $\sqrt{2x+5}+3=0$

$\sqrt{2x+5}=-3$, impossible

No solution

169. $\sqrt{x+1}-3x=1$

$\sqrt{x+1}=3x+1$

$x+1=9x^2+6x+1$

$0=9x^2+5x$

$0=x(9x+5)$

$x=0$

$9x+5=0 \implies x=-\frac{5}{9}$, extraneous

170. $\sqrt{x+5}-2x=3$

$\sqrt{x+5}=2x+3$

$x+5=4x^2+12x+9$

$4x^2+11x+4=0$

$$x=\frac{-11\pm\sqrt{121-64}}{8}=\frac{-11\pm\sqrt{57}}{8}$$

171. $\sqrt[3]{2x+1}+8=0$

$\sqrt[3]{2x+1}=-8$

$2x+1=-512$

$2x=-513$

$x=-\frac{513}{2}=-256.5$

172. $\sqrt[3]{4x-3}+2=0$

$(4x-3)^{1/3}=-2$

$4x-3=-8$

$4x=-5$

$x=-\frac{5}{4}$

173. $\sqrt{x}-\sqrt{x-5}=1$

$\sqrt{x}=1+\sqrt{x-5}$

$(\sqrt{x})^2=(1+\sqrt{x-5})^2$

$x=1+2\sqrt{x-5}+x-5$

$4=2\sqrt{x-5}$

$2=\sqrt{x-5}$

$4=x-5$

$9=x$

174. $\sqrt{x}+\sqrt{x-20}=10$

$\sqrt{x}=10-\sqrt{x-20}$

$(\sqrt{x})^2=(10-\sqrt{x-20})^2$

$x=100-20\sqrt{x-20}+x-20$

$-80=-20\sqrt{x-20}$

$4=\sqrt{x-20}$

$16=x-20$

$36=x$

175. $(x-5)^{2/3}=16$

$x-5=\pm16^{3/2}$

$x-5=\pm64$

$x=69, -59$

176. $(x^2-x-22)^{4/3}=16$

$x^2-x-22=\pm16^{3/4}$

$x^2-x-22=\pm8$

$x^2-x-30=0 \implies x=-5, 6$

$x^2-x-14=0 \implies x=\frac{1\pm\sqrt{57}}{2}$

177. $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

$$(x-1)^{1/2}[3x + 2(x-1)] = 0$$
$$(x-1)^{1/2}(5x-2) = 0$$
$$(x-1)^{1/2} = 0 \implies x - 1 = 0 \implies x = 1$$
$5x - 2 = 0 \implies x = \frac{2}{5}$ which is extraneous.

178. $4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} = 0$

$$2x[2x(x-1)^{1/3} + 3(x-1)^{4/3}] = 0$$
$$2x(x-1)^{1/3}[2x + 3(x-1)] = 0$$
$$2x(x-1)^{1/3}(5x-3) = 0$$
$$2x = 0 \implies x = 0$$
$$x - 1 = 0 \implies x = 1$$
$$5x - 3 = 0 \implies x = \frac{3}{5}$$

179. $\frac{1}{x} - \frac{1}{x+1} = 3$

$$x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} = x(x+1)(3)$$
$$x + 1 - x = 3x(x+1)$$
$$1 = 3x^2 + 3x$$
$0 = 3x^2 + 3x - 1$; $a = 3$, $b = 3$, $c = -1$
$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$$

180. $\frac{x}{x^2-4} + \frac{1}{x+2} = 3$

$$(x+2)(x-2)\frac{x}{x^2-4} + (x+2)(x-2)\frac{1}{x+2} = 3(x+2)(x-2)$$
$$x + x - 2 = 3x^2 - 12$$
$$3x^2 - 2x - 10 = 0$$
$a = 3$, $b = -2$, $c = -10$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)} = \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3}$$

181. $x = \frac{3}{x} + \frac{1}{2}$

$$(2x)(x) = (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right)$$
$$2x^2 = 6 + x$$
$$2x^2 - x - 6 = 0$$
$$(2x+3)(x-2) = 0$$
$$2x + 3 = 0 \implies x = -\frac{3}{2}$$
$$x - 2 = 0 \implies x = 2$$

182. $4x + 1 = \frac{3}{x}$

$$(x)4x + (x)1 = (x)\frac{3}{x}$$
$$4x^2 + x = 3$$
$$4x^2 + x - 3 = 0$$
$$(4x-3)(x+1) = 0$$
$$4x - 3 = 0 \implies x = \frac{3}{4}$$
$$x + 1 = 0 \implies x = -1$$

183. $|2x - 1| = 5$

$$2x - 1 = 5 \implies x = 3$$
$$-(2x-1) = 5 \implies x = -2$$

184. $|3x + 2| = 7$

$$3x + 2 = 7 \implies x = \frac{5}{3}$$
$$-(3x+2) = 7$$
$$-3x - 2 = 7 \implies x = -3$$

185. $|x| = x^2 + x - 3$

$x = x^2 + x - 3$ OR $-x = x^2 + x - 3$

$x^2 - 3 = 0$ $\qquad$ $x^2 + 2x - 3 = 0$

$x = \pm\sqrt{3}$ $\qquad$ $(x - 1)(x + 3) = 0$

$x - 1 = 0 \implies x = 1$

$x + 3 = 0 \implies x = -3$

Only $x = \sqrt{3}$, and $x = -3$ are solutions to the original equation. $x = -\sqrt{3}$ and $x = 1$ are extraneous. Note that the graph of $y = x^2 + x - 3 - |x|$ has two x-intercepts.

186. $|x - 10| = x^2 - 10x$

First equation:

$x - 10 = x^2 - 10x$

$0 = x^2 - 11x + 10$

$0 = (x - 1)(x - 10)$

$0 = x - 1 \implies x = 1$, not a solution

$0 = x - 10 \implies x = 10$

Second equation:

$-(x - 10) = x^2 - 10x$

$0 = x^2 - 9x - 10$

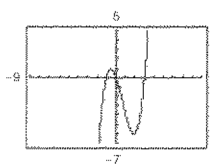
$0 = (x - 10)(x + 1)$

$0 = x - 10 \implies x = 10$

$0 = x + 1 \implies x = -1$

187. $y = x^3 - 2x^2 - 3x$

(a)



(b) x-intercepts: $(-1, 0), (0, 0), (3, 0)$

(c) $0 = x^3 - 2x^2 - 3x$

$0 = x(x + 1)(x - 3)$

$x = 0$

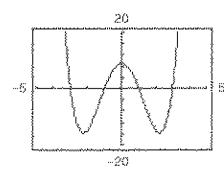
$x + 1 = 0 \implies x = -1$

$x - 3 = 0 \implies x = 3$

(d) The x-intercepts are the same as the solutions.

188. $y = x^4 - 10x^2 + 9$

(a)



(b) x-intercepts: $(\pm 1, 0), (\pm 3, 0)$

(c) $0 = x^4 - 10x^2 + 9$

$0 = (x^2 - 1)(x^2 - 9)$

$0 = (x + 1)(x - 1)(x + 3)(x - 3)$

$x + 1 = 0 \implies x = -1$

$x - 1 = 0 \implies x = 1$

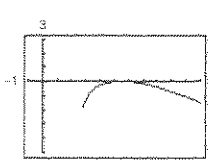
$x + 3 = 0 \implies x = -3$

$x - 3 = 0 \implies x = 3$

(d) The x-intercepts are the same as the solutions.

189. $y = \sqrt{11x - 30} - x$

(a)



(b) x-intercepts: $(5, 0), (6, 0)$

—CONTINUED—

189. **—CONTINUED—**

(c) $0 = \sqrt{11x - 30} - x$

$x = \sqrt{11x - 30}$

$x^2 = 11x - 30$

$x^2 - 11x + 30 = 0$

$(x - 5)(x - 6) = 0$

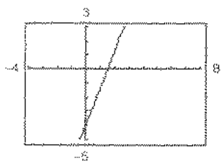
$x - 5 = 0 \implies x = 5$

$x - 6 = 0 \implies x = 6$

(d) The x-intercepts and the solutions are the same.

190. $y = 2x - \sqrt{15 - 4x}$

(a)



(b) x-intercept: $\left(\frac{3}{2}, 0\right)$

(c) $0 = 2x - \sqrt{15 - 4x}$

$\sqrt{15 - 4x} = 2x$

$15 - 4x = 4x^2$

$0 = 4x^2 + 4x - 15$

$0 = (2x + 5)(2x - 3)$

$0 = 2x + 5 \implies x = -\dfrac{5}{2}$

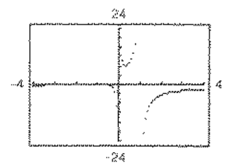
$0 = 2x - 3 \implies x = \dfrac{3}{2}$

$x = -\frac{5}{2}$ is extraneous. The x-intercept is $\left(\frac{3}{2}, 0\right)$.

(d) Same as intercept

191. $y = \dfrac{1}{x} - \dfrac{4}{x - 1} - 1$

(a)



(b) x-intercept: $(-1, 0)$

(c) $0 = \dfrac{1}{x} - \dfrac{4}{x - 1} - 1$

$0 = (x - 1) - 4x - x(x - 1)$

$0 = x - 1 - 4x - x^2 + x$

$0 = -x^2 - 2x - 1$

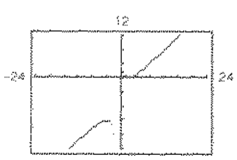
$0 = x^2 + 2x + 1$

$x + 1 = 0 \implies x = -1$

(d) The x-intercepts and the solutions are the same.

192. $y = x + \dfrac{9}{x + 1} - 5$

(a)



(b) x-intercept: $(2, 0)$

(d) Same as intercept

(c) $0 = x + \dfrac{9}{x + 1} - 5$

$0 = x(x + 1) + (x + 1)\dfrac{9}{x + 1} - 5(x + 1)$

$0 = x^2 + x + 9 - 5x - 5$

$0 = x^2 - 4x + 4$

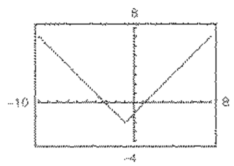
$0 = (x - 2)(x - 2)$

$0 = x - 2 \implies x = 2$

x-intercept: $(2, 0)$

193. $y = |x + 1| - 2$

(a)



(b) x-intercept: $(1, 0), (-3, 0)$

(c) $0 = |x + 1| - 2$

$2 = |x + 1|$

$x + 1 = 2$ or $-(x + 1) = 2$

$x = 1$ or $-x - 1 = 2$

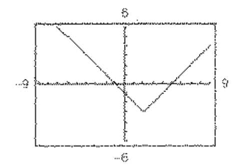
$-x = 3$

$x = -3$

(d) The x-intercepts and the solutions are the same.

194. $y = |x - 2| - 3$

(a)



(b) x-intercepts: $(5, 0), (-1, 0)$

(c) $0 = |x - 2| - 3$

$3 = |x - 2|$

First equation:

$x - 2 = 3 \Longrightarrow x = 5$

Second equation:

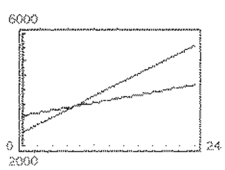
$-(x - 2) = 3$

$-x + 2 = 3 \Longrightarrow x = -1$

x-intercepts: $(5, 0), (-1, 0)$

(d) Same as intercepts

195. (a)



Intersection: $(6.7, 3388.7)$

(c) The slopes indicate the change in population per year. Arizona's population is growing faster.

(b) $45.2t + 3087 = 128.2t + 2533$

$83t = 554$

$t \approx 6.7$

$A = S \approx 3388.7$

The point $(6.7, 3388.7)$ indicates the year, 1986, in which the two populations were the same, about 3388.7 thousand.

(d) For 2010, $t = 30$ and $S \approx 4443$ thousand and $A \approx 6379$ thousand. Answers will vary.

196. (a) $P = 0.1220t^2 + 1.529t + 18.72 = 40$

$0.122t^2 + 1.529t - 21.28 = 0$

$$t = \frac{-1.529 \pm \sqrt{(1.529)^2 - 4(0.122)(-21.28)}}{2(0.122)}$$

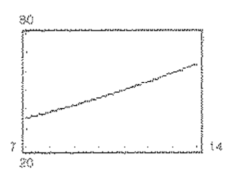
$$= \frac{-1.529 \pm \sqrt{12.7225}}{0.244}$$

$t \approx 8.35, -20.89$

Taking the positive root, $t \approx 8.35$ or 1998. Similarly, $P = 50$ yields 10.93, or 2000.

(b) Answers will vary.

(c)

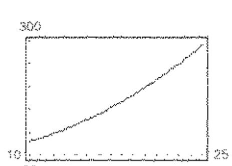


(d) $P = 75$ when $t \approx 16.1$, or 2006.

(e) Answers will vary.

197. $C = 0.45x^2 - 1.65x + 50.75,\ 10 \le x \le 25$

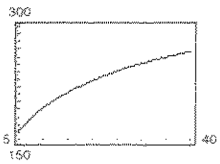
(a)



(b) If $C = 150$, then $x = 16.797$ degrees.

(c) If the temperature is increased $10°$ to $20°$, then C increases from 79.25 to 197.75, a factor of 2.5.

198. $T = 75.82 - 2.11x + 43.51\sqrt{x}, 5 \le x \le 40$

(a)


(b) For $x = 14.696$, $T \approx 211.6$°F.

(c) For $T = 240$°F, $x \approx 24.725$ pounds/in.^{2}.

199. False. Two linear equations could have an infinite number of points of intersection. For example, $x + y = 1$ and $2x + 2y = 2$.

200. False. An equation can have any number of extraneous solutions. For example, $|x| = x^2 + x + 3$.

201.
$$\begin{aligned} 2x - 5c &= 10 + 3c - 3x, \quad x = 3 \\ 2(3) - 5c &= 10 + 3c - 3(3) \\ 6 - 5c &= 1 + 3c \\ 5 &= 8c \\ c &= \tfrac{5}{8} \end{aligned}$$

202.
$$\begin{aligned} 5x + 2c &= 12 + 4x - 2c, \quad x = 2 \\ 5(2) + 2c &= 12 + 4(2) - 2c \\ 10 + 2c &= 20 - 2c \\ 4c &= 10 \\ c &= \tfrac{5}{2} \end{aligned}$$

203. (a)
$$\begin{aligned} ax^2 + bx &= 0 \\ x(ax + b) &= 0 \\ x &= 0 \\ x &= -b/a \end{aligned}$$

(b)
$$\begin{aligned} ax^2 - ax &= 0 \\ ax(x - 1) &= 0 \\ x &= 0 \\ x &= 1 \end{aligned}$$

Appendix B.4 Solving Inequalities Algebraically and Graphically

- You should be able to solve an inequality algebraically using the Properties of Inequalities.
- You should be able to solve inequalities involving absolute values.
- You should be able to solve polynomial inequalities using critical numbers and test intervals.
- You should be able to solve rational inequalities.
- You should be able to solve inequalities using a graphing utility.

Vocabulary Check

1. negative
2. double
3. $-a \le x \le a$
4. $x \le -a, x \ge a$
5. zeros, undefined values

1. $x < 3$

Matches (f).

2. $x \ge 5$

Matches (a).

3. $-3 < x \le 4$

Matches (d).

4. $0 \le x \le \frac{9}{2}$

Matches (b).

5. $-1 \le x \le \frac{5}{2}$

Matches (e).

6. $-1 < x < \frac{5}{2}$

Matches (c).

7. (a) $x = 3$

$5(3) - 12 \overset{?}{>} 0$

$3 > 0$

Yes, $x = 3$ is a solution.

(b) $x = -3$

$5(-3) - 12 \overset{?}{>} 0$

$-27 \not> 0$

No, $x = -3$ is not a solution.

(c) $x = \frac{5}{2}$

$5\left(\frac{5}{2}\right) - 12 \overset{?}{>} 0$

$\frac{1}{2} > 0$

Yes, $x = \frac{5}{2}$ is a solution.

(d) $x = \frac{3}{2}$

$5\left(\frac{3}{2}\right) - 12 \overset{?}{>} 0$

$-\frac{9}{2} \not> 0$

No, $x = \frac{3}{2}$ is not a solution.

8. $-5 < 2x - 1 \le 1$

(a) $x = -\frac{1}{2}$

$-5 \overset{?}{<} 2\left(-\frac{1}{2}\right) - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} -1 - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} -2 \overset{?}{\le} 1$

Yes, $x = -\frac{1}{2}$ is a solution.

(b) $x = -\frac{5}{2}$

$-5 \overset{?}{<} 2\left(-\frac{5}{2}\right) - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} -6 \overset{?}{\le} 1$

No, $x = -\frac{5}{2}$ is not a solution.

(c) $x = \frac{4}{3}$

$-5 \overset{?}{<} 2\left(\frac{4}{3}\right) - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} \frac{8}{3} - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} \frac{5}{3} \overset{?}{\le} 1$

No, $x = \frac{4}{3}$ is not a solution.

(d) $x = 0$

$-5 \overset{?}{<} 2(0) - 1 \overset{?}{\le} 1$

$-5 < -1 \overset{?}{\le} 1$

Yes, $x = 0$ is a solution.

9. $-1 < \dfrac{3 - x}{2} \le 1$

(a) $x = 0$

$-1 \overset{?}{<} \dfrac{3 - 0}{2} \overset{?}{\le} 1$

$-1 \overset{?}{<} \dfrac{3}{2} \overset{?}{\le} 1$

No, $x = 0$ is not a solution.

(b) $x = \sqrt{5}$

$-1 \overset{?}{<} \dfrac{3 - \sqrt{5}}{2} \overset{?}{\le} 1$

$-1 \overset{?}{<} 0.382 \overset{?}{\le} 1$

Yes, $x = \sqrt{5}$ is a solution.

(c) $x = 1$

$-1 \overset{?}{<} \dfrac{3 - 1}{2} \overset{?}{\le} 1$

$-1 \overset{?}{<} 1 \overset{?}{\le} 1$

Yes, $x = 1$ is a solution.

(d) $x = 5$

$-1 \overset{?}{<} \dfrac{3 - 5}{2} \le 1$

$-1 \overset{?}{<} -1 \overset{?}{\le} 1$

No, $x = 5$ is not a solution.

10. $|x - 10| \geq 3$

(a) $x = 13$

$|13 - 10| \stackrel{?}{\geq} 3$

$3 \geq 3$

Yes, $x = 13$ is a solution.

(b) $x = -1$

$|-1 - 10| \stackrel{?}{\geq} 3$

$|-11| \stackrel{?}{\geq} 3$

$11 \geq 3$

Yes, $x = -1$ is a solution.

(c) $x = 14$

$|14 - 10| \stackrel{?}{\geq} 3$

$4 \geq 3$

Yes, $x = 14$ is a solution.

(d) $x = 9$

$|9 - 10| \stackrel{?}{\geq} 3$

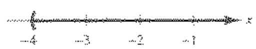
$1 \stackrel{?}{\geq} 3$

No, $x = 9$ is not a solution.

11. $-10x < 40$

$-\frac{1}{10}(-10x) > -\frac{1}{10}(40)$

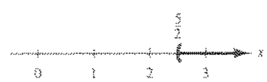
$x > -4$



12. $6x > 15$

$2x > 5$

$x > \frac{5}{2}$

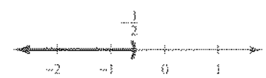


13. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

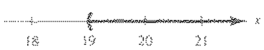
$x < -\frac{1}{2}$



14. $2x + 7 < 3(x - 4)$

$2x + 7 < 3x - 12$

$19 < x$

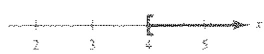


15. $\frac{3}{4}x - 6 \leq x - 7$

$1 \leq \frac{1}{4}x$

$4 \leq x$

$x \geq 4$



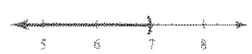
16. $3 + \frac{2}{7}x > x - 2$

$21 + 2x > 7x - 14$

$35 > 5x$

$7 > x$

$x < 7$



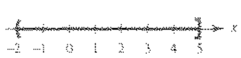
17. $-8 \leq 1 - 3(x - 2) < 13$

$-8 \leq 1 - 3x + 6 < 13$

$-8 \leq -3x + 7 < 13$

$-15 \leq -3x < 6$

$5 \geq x > -2 \implies -2 < x \leq 5$

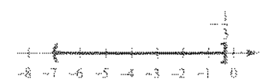


18. $0 \leq 2 - 3(x + 1) < 20$

$0 \leq -3x - 1 < 20$

$1 \leq -3x < 21$

$-\frac{1}{3} \geq x > -7$

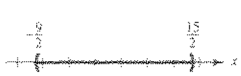


19. $-4 < \dfrac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

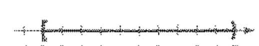
$-\dfrac{9}{2} < x < \dfrac{15}{2}$



20. $0 \leq \dfrac{x + 3}{2} < 5$

$0 \leq x + 3 < 10$

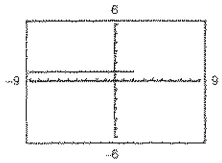
$-3 \leq x < 7$



21. $5 - 2x \geq 1$

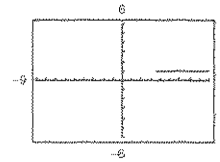
$-2x \geq -4$

$x \leq 2$



22. $20 < 6x - 1$

$x > \frac{7}{2}$

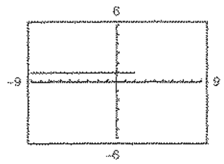


23. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

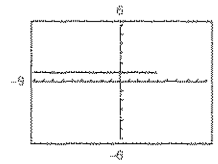
$2x < 4$

$x < 2$

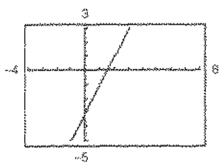


24. $4(x - 3) \leq 8 - x$

$x \leq 4$



25.



Using the graph, (a) $y \geq 1$ for $x \geq 2$ and (b) $y \leq 0$ for $x \leq \frac{3}{2}$.
Algebraically:

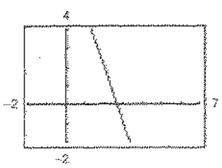
(a) $y \geq 1$ | (b) $y \leq 0$

$2x - 3 \geq 1$ | $2x - 3 \leq 0$

$2x \geq 4$ | $2x \leq 3$

$x \geq 2$ | $x \leq \frac{3}{2}$

26.



Using the graph, (a) $-1 \leq y \leq 3$ for $\frac{5}{3} \leq x \leq 3$, and (b) $y \leq 0$ for $x \geq \frac{8}{3}$.
Algebraically:

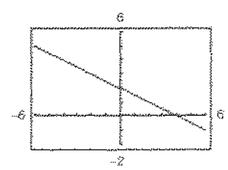
(a) $-1 \leq y \leq 3$ | (b) $y \leq 0$

$-1 \leq -3x + 8 \leq 3$ | $-3x + 8 \leq 0$

$-9 \leq -3x \leq -5$ | $8 \leq 3x$

$3 \geq x \geq \frac{5}{3}$ | $\frac{8}{3} \leq x$

27.



Using the graph, (a) $0 \leq y \leq 3$ for $-2 \leq x \leq 4$ and (b) $y \geq 0$ for $x \leq 4$.
Algebraically:

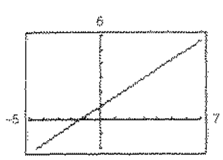
(a) $0 \leq y \leq 3$ | (b) $y \geq 0$

$0 \leq -\frac{1}{2}x + 2 \leq 3$ | $-\frac{1}{2}x + 2 \geq 0$

$-2 \leq -\frac{1}{2}x \leq 1$ | $2 \geq \frac{1}{2}x$

$4 \geq x \geq -2$ | $4 \geq x$

28.



Using the graph, (a) $y \leq 5$ for $x \leq 6$, and (b) $y \geq 0$ for $x \geq -\frac{3}{2}$.
Algebraically:

(a) $y \leq 5$ | (b) $y \geq 0$

$\frac{2}{3}x + 1 \leq 5$ | $\frac{2}{3}x + 1 \geq 0$

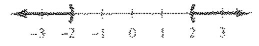
$\frac{2}{3}x \leq 4$ | $\frac{2}{3}x \geq -1$

$x \leq 6$ | $x \geq -\frac{3}{2}$

29. $|5x| > 10$

$5x < -10$ or $5x > 10$

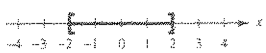
$x < -2$ or $x > 2$



30. $\left|\dfrac{x}{2}\right| \le 1$

$|x| \le 2$

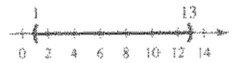
$-2 \le x \le 2$



31. $|x - 7| < 6$

$-6 < x - 7 < 6$

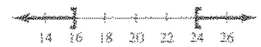
$1 < x < 13$



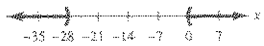
32. $|x - 20| \ge 4$

$x - 20 \ge 4 \quad \text{or} \quad x - 20 \le -4$

$x \ge 24 \quad \text{or} \quad x \le 16$



33. $|x + 14| + 3 > 17$

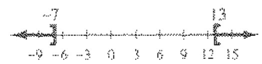


$|x + 14| > 14$

$x + 14 < -14 \quad \text{or} \quad x + 14 > 14$

$x < -28 \quad \text{or} \quad x > 0$

34. $\left|\dfrac{x-3}{2}\right| \ge 5$

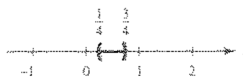


$|x - 3| \ge 10$

$x - 3 \ge 10 \quad \text{or} \quad x - 3 \le -10$

$x \ge 13 \quad \text{or} \quad x \le -7$

35. $10|1 - 2x| < 5$



$|1 - 2x| < \frac{1}{2}$

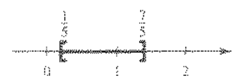
$-\frac{1}{2} < 1 - 2x < \frac{1}{2}$

$-\frac{3}{2} < -2x < -\frac{1}{2}$

$\frac{3}{4} > x > \frac{1}{4}$

$\frac{1}{4} < x < \frac{3}{4}$

36. $3|4 - 5x| \le 9$



$|4 - 5x| \le 3$

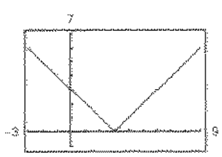
$-3 \le 4 - 5x \le 3$

$-7 \le -5x \le -1$

$\frac{7}{5} \ge x \ge \frac{1}{5}$

$\frac{1}{5} \le x \le \frac{7}{5}$

37. $y = |x - 3|$



Graphically, (a) $y \le 2$ for $1 \le x \le 5$ and (b) $y \ge 4$ for $x \le -1$ or $x \ge 7$.

Algebraically:

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$

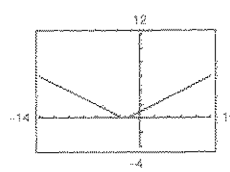
(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4 \quad \text{or} \quad x - 3 \ge 4$

$x \le -1 \quad \text{or} \quad x \ge 7$

38. $y = \left|\frac{1}{2}x + 1\right|$



Algebraically:

(a) $y \le 4$

$\left|\frac{1}{2}x + 1\right| \le 4$

$-4 \le \frac{1}{2}x \le 4$

$-5 \le \frac{1}{2}x \le 3$

$-10 \le x \le 6$

(b) $y \ge 1$

$\left|\frac{1}{2}x + 1\right| \ge 1$

$\frac{1}{2}x + 1 \le -1 \quad \text{or} \quad \frac{1}{2}x + 1 \ge 1$

$\frac{1}{2}x \le -2 \quad \text{or} \quad \frac{1}{2}x \ge 0$

$x \le -4 \quad \text{or} \quad x \ge 0$

39. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than three units from 0.

$|x - 0| \leq 3$

$|x| \leq 3$

40. The graph shows all real numbers no more than four units from -1.

$|x + 1| \leq 4$

41. The midpoint of the interval $[-3, 3]$ is 0. The two intervals represent all numbers x more than three units from 0.

$|x - 0| > 3$

$|x| > 3$

42. The graph shows all real numbers at least three units from 7.

$|x - 7| \geq 3$

43. All real numbers within 10 units of 7

$|x - 7| \leq 10$

44. $|x + 5| \leq 8$

45. All real numbers at least five units from 3

$|x - 3| \geq 5$

46. All real numbers more than three units from -1

$|x + 1| > 3$

47. $x^2 - 4x - 5 > 0$

$(x - 5)(x + 1) > 0$

Critical numbers: $-1, 5$

Testing the intervals $(-\infty, -1)$, $(-1, 5)$ and $(5, \infty)$, we have $x^2 - 4x - 5 > 0$ on $(-\infty, -1)$ and $(5, \infty)$. Similarly, $x^2 - 4x - 5 < 0$ on $(-1, 5)$.

48. $x^2 - 3x - 4 > 0$

$(x - 4)(x + 1) > 0$

Critical numbers: $-1, 4$

Testing the intervals $(-\infty, -1)$, $(-1, 4)$, and $(4, \infty)$, we have $x^2 - 3x - 4 > 0$ on $(-\infty, -1)$ and $(4, \infty)$. Similarly, $x^2 - 3x - 4 < 0$ on $(-1, 4)$.

49. $2x^2 - 4x - 3 = 0$

$$x = \frac{4 \pm \sqrt{16 + 24}}{4} = 1 \pm \frac{\sqrt{10}}{2}$$

Entirely negative: $\left(1 - \frac{\sqrt{10}}{2}, 1 + \frac{\sqrt{10}}{2}\right) \approx (-0.581, 2.581)$

Entirely positive: $\left(-\infty, 1 - \frac{\sqrt{10}}{2}\right) \cup \left(1 + \frac{\sqrt{10}}{2}, \infty\right)$

50. $2x^2 - x - 5 = 0$

$$x = \frac{1 \pm \sqrt{1 - 4(2)(-5)}}{2(2)} = \frac{1 \pm \sqrt{41}}{4}, \text{ critical numbers}$$

Test intervals: $\left(-\infty, \frac{1}{4} - \frac{\sqrt{41}}{4}\right)$, $\left(\frac{1}{4} - \frac{\sqrt{41}}{4}, \frac{1}{4} + \frac{\sqrt{41}}{4}\right)$, and $\left(\frac{1}{4} + \frac{\sqrt{41}}{4}, \infty\right)$

Testing these intervals, we have

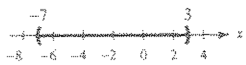
$2x^2 - x - 5 > 0$ on $\left(-\infty, \frac{1}{4} - \frac{\sqrt{41}}{4}\right)$ and $\left(\frac{1}{4} + \frac{\sqrt{41}}{4}, \infty\right)$,

$2x^2 - x - 5 < 0$ on $\left(\frac{1}{4} - \frac{\sqrt{41}}{4}, \frac{1}{4} + \frac{\sqrt{41}}{4}\right)$.

51. $x^2 - 4x + 5 > 0$ for all x. There are no critical numbers because $x^2 - 4x + 5 \neq 0$. The only test interval is $(-\infty, \infty)$.

52. $-x^2 + 6x - 10 < 0$ for all x. There are no critical numbers because $-x^2 + 6x - 10 \neq 0$. The only test interval is $(-\infty, \infty)$.

53. $(x+2)^2 < 25$



$x^2 + 4x + 4 < 25$

$x^2 + 4x - 21 < 0$

$(x+7)(x-3) < 0$

Critical numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

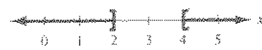
Test: Is $(x+7)(x-3) < 0$?

Solution set: $(-7, 3)$

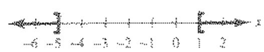
54. $(x-3)^2 \geq 1$

$x - 3 \geq 1$ or $x - 3 \leq -1$

$x \geq 4$ or $x \leq 2$



55. $x^2 + 4x + 4 \geq 9$



$x^2 + 4x - 5 \geq 0$

$(x+5)(x-1) \geq 0$

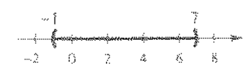
Critical numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x+5)(x-1) \geq 0$?

Solution set: $(-\infty, -5] \cup [1, \infty)$

56. $x^2 - 6x + 9 < 16$



$x^2 - 6x - 7 < 0$

$(x+1)(x-7) < 0$

Critical numbers: $x = -1, x = 7$

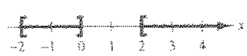
Test intervals: $(-\infty, -1) \implies (x+1)(x-7) > 0$

$(-1, 7) \implies (x+1)(x-7) < 0$

$(7, \infty) \implies (x+1)(x-7) > 0$

Solution interval: $(-1, 7)$

57. $x^3 - 4x \geq 0$



$x(x+2)(x-2) \geq 0$

Critical number: $x = 0, x = \pm 2$

Test intervals: $(-\infty, -2), (-2, 0), (0, 2), (2, \infty)$

Test: Is $x(x+2)(x-2) \geq 0$?

Solution set: $[-2, 0] \cup [2, \infty)$

58. $x^4(x-3) \leq 0$

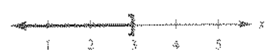
Critical numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \implies x^4(x-3) < 0$

$(0, 3) \implies x^4(x-3) < 0$

$(3, \infty) \implies x^4(x-3) > 0$

Solution intervals: $(-\infty, 0] \cup [0, 3]$ or $(-\infty, 3]$



59. $3x^2 - 11x + 16 \leq 0$

Since $b^2 - 4ac = -71 < 0$, there are no real solutions to $3x^2 - 11x + 16 = 0$. In fact, $3x^2 - 11x + 16 > 0$ for all x.

No solution

60. $4x^2 + 12x + 9 \leq 0$

$(2x+3)^2 \leq 0$

$2x + 3 = 0$

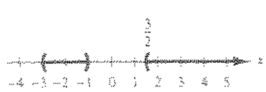
$x = -\frac{3}{2}$

61. $2x^3 + 5x^2 - 6x - 9 > 0$

$(x+1)(x+3)(2x-3) > 0$

Critical numbers: $-3, -1, \frac{3}{2}$

Testing the four intervals, we see that $2x^3 + 5x^2 - 6x - 9 > 0$ on $(-3, -1)$ and $\left(\frac{3}{2}, \infty\right)$.

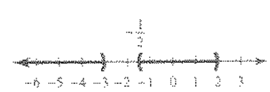


62. $2x^3 + 3x^2 - 11x - 6 < 0$

$(x-2)(x+3)(2x+1) < 0$

Critical numbers: $-3, -\frac{1}{2}, 2$

Testing the four intervals, we see that $2x^3 + 3x^2 - 11x - 6 < 0$ on $(-\infty, -3)$ and $\left(-\frac{1}{2}, 2\right)$.



63. (a) $f(x) = g(x)$ when $x = 1$.

(b) $f(x) \geq g(x)$ when $x \geq 1$.

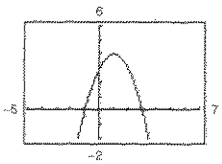
(c) $f(x) > g(x)$ when $x > 1$.

64. (a) $f(x) = g(x)$ when $x = -1$ or $x = 3$.

(b) $f(x) \geq g(x)$ when $x \leq -1$ or $x \geq 3$.

(c) $f(x) > g(x)$ when $x < -1$ or $x > 3$.

65. $y = -x^2 + 2x + 3$



(a) $y \leq 0$ when $x \leq -1$ or $x \geq 3$.

(b) $y \geq 3$ when $0 \leq x \leq 2$.

Algebraically,

$$-x^2 + 2x + 3 \leq 0$$
$$x^2 - 2x - 3 \geq 0$$
$$(x - 3)(x + 1) \geq 0$$

Critical numbers: $x = -1, x = 3$

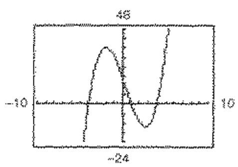
Testing the intervals $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$, you obtain $x \leq -1$ or $x \geq 3$.

$$-x^2 + 2x + 3 \geq 3$$
$$-x^2 + 2x \geq 0$$
$$x^2 - 2x \leq 0$$
$$x(x - 2) \leq 0$$

Critical numbers: $x = 0, x = 2$

Testing the intervals $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$, you obtain $0 \leq x \leq 2$.

66. $y = x^3 - x^2 - 16x + 16$



(a) $y \leq 0$

$$x^3 - x^2 - 16x + 16 \leq 0$$
$$x^2(x - 1) - 16(x - 1) \leq 0$$
$$(x - 1)(x^2 - 16) \leq 0$$

$y \leq 0$ when $-\infty < x \leq -4, 1 \leq x \leq 4$.

(b) $y \geq 36$

$$x^3 - x^2 - 16x + 16 \geq 36$$
$$x^3 - x^2 - 16x - 20 \geq 0$$
$$(x + 2)(x - 5)(x + 2) \geq 0$$

$y \geq 36$ when $x = -2, 5 \leq x < \infty$.

67. $\dfrac{1}{x} - x > 0$

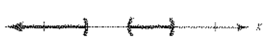
$$\frac{1 - x^2}{x} > 0$$

Critical numbers: $x = 0, x = \pm 1$

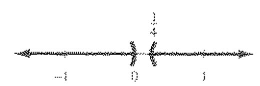
Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Test: Is $\dfrac{1 - x^2}{x} > 0$?

Solution set: $(-\infty, -1) \cup (0, 1)$



68. $\dfrac{1}{x} - 4 < 0$



$$\frac{1 - 4x}{x} < 0$$

Critical numbers: $x = 0, x = \dfrac{1}{4}$

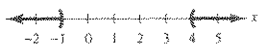
Test intervals: $(-\infty, 0) \implies \dfrac{1 - 4x}{x} < 0$

$$\left(0, \frac{1}{4}\right) \implies \frac{1 - 4x}{x} > 0$$

$$\left(\frac{1}{4}, \infty\right) \implies \frac{1 - 4x}{x} < 0$$

Solution interval: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$

69. $$\frac{x+6}{x+1} - 2 < 0$$



$$\frac{x+6-2(x+1)}{x+1} < 0$$

$$\frac{4-x}{x+1} < 0$$

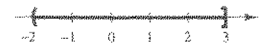
Critical numbers: $x = -1, x = 4$

Test intervals: $(-\infty, -1), (-1, 4), (4, \infty)$

Test: Is $\frac{4-x}{x+1} < 0$?

Solution set: $(-\infty, -1) \cup (4, \infty)$

70. $$\frac{x+12}{x+2} - 3 \geq 0$$



$$\frac{x+12-3(x+2)}{x+2} \geq 0$$

$$\frac{6-2x}{x+2} \geq 0$$

Critical numbers: $x = -2, x = 3$

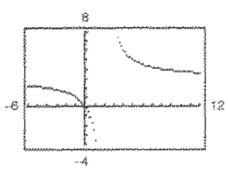
Test intervals: $(-\infty, -2) \implies \frac{6-2x}{x+2} < 0$

$(-2, 3) \implies \frac{6-2x}{x+2} > 0$

$(3, \infty) \implies \frac{6-2x}{x+2} < 0$

Solution interval: $(-2, 3]$

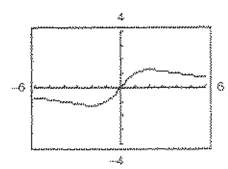
71. $y = \frac{3x}{x-2}$



(a) $y \leq 0$ when $0 \leq x < 2$.

(b) $y \geq 6$ when $2 < x \leq 4$.

72. $y = \frac{5x}{x^2+4}$



(a) $$y \geq 1$$

$$\frac{5x}{x^2+4} \geq 1$$

$$\frac{5x-(x^2+4)}{(x^2+4)} \geq 0$$

$$\frac{(x-4)(x-1)}{x^2+4} \leq 0$$

$y \geq 1$ when $1 \leq x \leq 4$.

(b) $$y \leq 0$$

$$\frac{5x}{x^2+4} \leq 0$$

$y \leq 0$ when $-\infty < x \leq 0$.

73. $\sqrt{x-5}$

Need: $x - 5 \geq 0$

$x \geq 5$

Domain: $[5, \infty)$

74. $\sqrt[4]{6x+15}$

$6x + 15 \geq 0$

$6x \geq -15$

$x \geq -\frac{15}{6}$

$x \geq -\frac{5}{2}$

$\left[-\frac{5}{2}, \infty\right)$

75. $\sqrt[3]{6-x}$

Domain: all real x

76. $\sqrt[3]{2x^2-8}$

Domain: all real x

77. $\sqrt{x^2 - 4}$

Need: $x^2 - 4 \geq 0$

$(x + 2)(x - 2) \geq 0$

$x \leq -2$ or $x \geq 2$

Domain: $(-\infty, -2] \cup [2, \infty)$

78. $\sqrt[4]{4 - x^2}$

Need: $4 - x^2 \geq 0$

$x^2 - 4 \leq 0$

$(x - 2)(x + 2) \leq 0$

Testing each interval, the solution is $-2 \leq x \leq 2$.

Domain: $[-2, 2]$

79. (a) $P(t) = 1000$

This occurs at the point of intersection, $t \approx 4$, or 1994.

(b) Less than one million: $P(t) < 1000$
This occurs for $t < 4$, or before 1994.

Greater than one million: $P(t) > 1000$
This occurs for $t > 4$, or after 1994.

80. (a) $p(t) = 2450$ for $t \approx 8$, or 1998

(b) $p(t) < 2450$ for (1998, 2004), and $p(t) > 2450$ for (1993, 1998).

81. (a) $s = -16t^2 + v_0t + s_0$

$s = -16t^2 + 160t$

$s = 16t(10 - t)$

$s = 0$ when $t = 10$ seconds.

(b) $s = -16t^2 + 160t > 384$

$16t^2 - 160t + 384 < 0$

$16(t - 6)(t - 4) < 0$

$s > 384$ when $4 < t < 6$.

82. (a) $s = -16t^2 + v_0t + s_0$

$= -16t^2 + 128t$

$= 16t(8 - t)$

$t = 8$ seconds

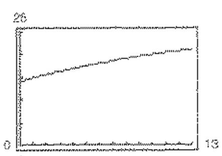
(b) $-16t^2 + 128t < 128$

$16t^2 - 128t + 128 < 0$

Critical numbers: 1.17, 6.83

(0, 1.17), (6.83, 8)

83. (a)



(b) $15 < D < 20$ for $1.28 < t < 10.09$, or between 1991 and 2000

(c) $15 < D < 20$

$15 < -0.0165t^2 + 0.755t + 14.06 < 20$

To solve these inequalities, find the critical numbers.

$0.0165t^2 - 0.755t + 0.94 = 0$

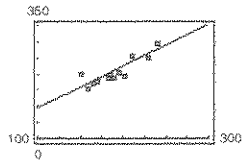
$$t = \frac{0.755 \pm \sqrt{(-0.755)^2 - 4(0.0165)(0.94)}}{2(0.0165)}$$

$$= \frac{0.755 \pm \sqrt{0.507985}}{0.033}$$

Because $0 < t < 13$, select the negative sign, $t \approx 1.28$. Hence, $15 < D$ for $1.28 < t$.
Similarly, $D < 20$ for $t < 10.09$.

(d) No. $D(t) < 30$ for all t.

84. (a), (b)



(c) For $y \geq 200$, $x \geq 181.5$ pounds.

(d) The model is not accurate. The data is not linear. Other factors include muscle strength, height, physical condition, etc.

85.
$$V(t) \geq 65$$
$$3.37t + 57.9 \geq 65$$
$$3.37t \geq 7.1$$
$$t \geq 2.11$$

The number of hours playing video games exceeded 65 in 2002.

86.
$$N(t) \leq 175$$
$$-2.51t + 179.6 \leq 175$$
$$4.6 \leq 2.51t$$
$$1.83 \leq t$$

The number of hours reading daily newspapers was below 175 at the end of 2001.

87.
$$V(t) = N(t)$$
$$3.37t + 57.9 = -2.51t + 179.6$$
$$5.88t = 121.7$$
$$t \approx 20.7$$

According to these models, the number of hours reading daily newspapers and playing video games will be the same in 2020.

88.
$$V(t) > N(t)$$
$$3.37t + 57.9 > -2.51t + 179.6$$
$$5.88t > 121.7$$
$$t > 20.7$$

According to these models, the number of hours playing video games will exceed the number of hours reading daily newspapers in 2020.

89. When $t = 2$, $v \approx 333$ vibrations per second.

90. For $v = 600$, $t \approx 3.6$ mm.

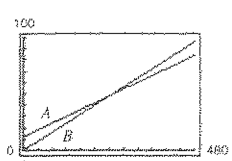
91. When $200 \leq v \leq 400$, $1.2 < t < 2.4$.

92. For $t < 3$, $0 < v < 500$.

93. (a) Option A: $A(t) = 0.15t + 12$

Option B: $B(t) = 0.20t$

(b)



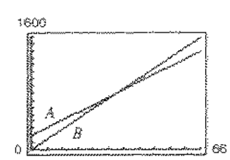
(c) $A(t) = B(t)$ when $t = 240$. $B(t)$ is the better choice if you use less than 240 minutes. $A(t)$ is the better choice if you use more than 240 minutes.

(d) Answers will vary.

94. (a) Option A: $A(t) = 18t + 200$

Option B: $B(t) = 24t$

(b)



(c) $A(t) = B(t)$ when $t \approx 33.3$. $B(t)$ is the better choice if the move takes less than 33.3 hours. $A(t)$ is the better choice if the move takes more than 33.3 hours.

(d) Answers will vary.

95. False. If $-10 \leq x \leq 8$, then $10 \geq -x$ and $-x \geq -8$.

96. True. $\frac{3}{2}x^2 + 3x + 6 \geq 0$ for all x.

97. The polynomial $f(x) = (x - a)(x - b)$ is zero at $x = a$ and $x = b$.

98. For $(-\infty, a)$, $(x - a) < 0$, $(x - b) < 0$, and $(x - a)(x - b) > 0$.

For (a, b), $(x - a) > 0$, $(x - b) < 0$, and $(x - a)(x - b) < 0$.

For (b, ∞), $(x - a) > 0$, $(x - b) > 0$, and $(x - a)(x - b) > 0$.

The polynomial changes signs at the zeros, $x = a$ and $x = b$.

99. (iv) $a < b$

(ii) $2a < 2b$

(iii) $2a < a + b < 2b$

(i) $a < \dfrac{a + b}{2} < b$

100. (ii) $0 < a < b$

(i) $a^2 < ab < b^2$

(iii) $a < \sqrt{ab} < b$

Appendix B.5 Representing Data Graphically

- You should be able to construct line plots.
- You should be able to construct histograms or frequency distributions.
- You should be able to construct bar graphs.
- You should be able to construct line graphs.

Vocabulary Check

1. Statistics **2.** Line plots **3.** histogram

4. frequency distribution **5.** bar graph **6.** Line graphs

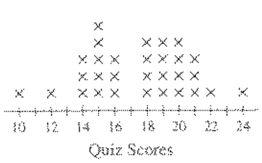
1. (a) The price 2.569 occurred with the greatest frequency (6).

(b) The prices range from 2.459 to 2.649. The range is $2.649 - 2.459 = 0.19$.

2. (a) The weight of 900 pounds occurred with the greatest frequency (9).

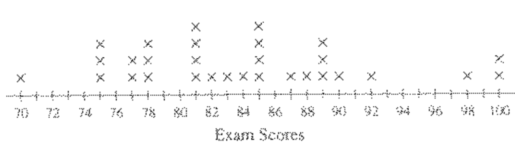
(b) The weights range from 600 to 1300 pounds. The range of weights is $1300 - 600 = 700$ pounds.

3.



The score of 15 occurred with the greatest frequency.

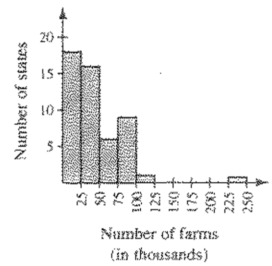
4.



The scores 81 and 85 occurred with the greatest frequency.

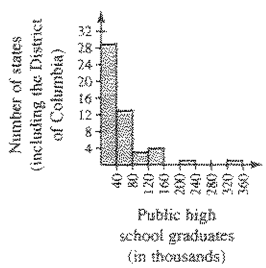
5. (Answers will depend on intervals selected.)

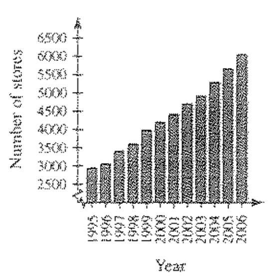
Interval	*Tally*
[0, 25)	𝍸 𝍸 𝍸 |||
[25, 50)	𝍸 𝍸 𝍸 |
[50, 75)	𝍸 |
[75, 100)	𝍸 |||
[100, 125)	|
[125, 150)	
[150, 175)	
[175, 200)	
[200, 225)	
[225, 250)	|



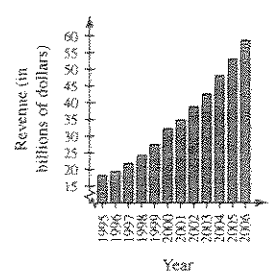
6. Sample answer:

Interval	*Tally*
[0, 40)	𝍸 𝍸 𝍸 𝍸 𝍸 ||||
[40, 80)	𝍸 𝍸 |||
[80, 120)	|||
[120, 160)	||||
[160, 200)	
[200, 240)	|
[240, 280)	
[280, 320)	
[320, 360)	|



7. 

From 1995 to 2006, the number of Wal-Mart stores increases at a fairly constant rate.

8. 

From 1995 to 2006, the revenue of Costco Wholesale increases every year. Moreover, the rate of increase appears to increase.

9. 1999: $13,428 - 2430 = \$10,998$

2000: $14,081 - 2506 = \$11,575$

2001: $15,000 - 2562 = \$12,438$

2002: $15,742 - 2700 = \$13,042$

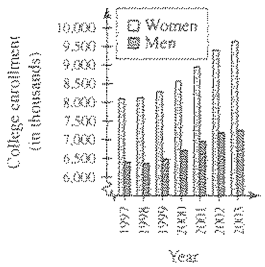
2003: $16,383 - 2903 = \$13,480$

2004: $17,442 - 3313 = \$14,129$

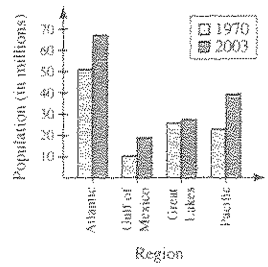
10.

	Public	*Private*
1999–2000	76	653
2000–2001	56	919
2001–2002	138	742
2002–2003	203	641
2003–2004	410	1059

11.



12.



13. From 1996 to 2005:

$$\frac{2400 - 1100}{1100} \approx 1.18, \text{ or } 118\%$$

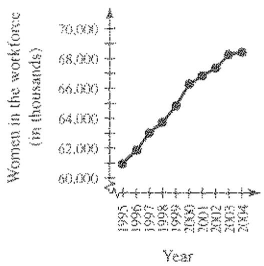
14. (a) 1995 to 1999: $1600 - 1180 = 420$, or \$420,000 increase

(b) 2000 to 2005: $2400 - 2200 = 200$, or \$200,000 increase

15. Highest price was \$2.59 in January.

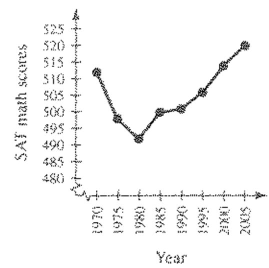
16. $2.59 - 2.46 = \$0.13$

17.



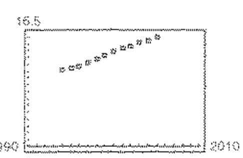
From 1995 to 2004, the total number of women in the work force increases at a fairly constant rate.

18.

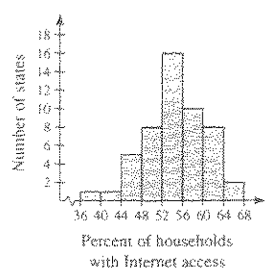


From 1970 to 1980, the SAT math scores decreased, and from 1980 to 2003 they increased.

19.

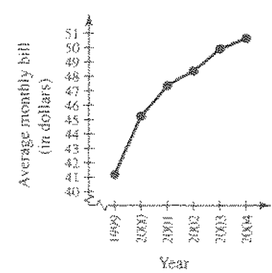


20.



Answers will vary.

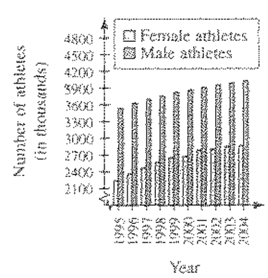
21.



Answers will vary.

22. In 1990, cell phone bills might have been high because they were still new and not so popular. In order for providers to make money, they had to charge higher prices than now.

23.



Answers will vary.

24. A histogram has a portion of the real number line as its horizontal axis, and the bars are not separated by spaces. A bar graph can be either horizontal or vertical. The labels are not necessarily numbers, and the bars are usually separated by spaces.

25. Answers will vary. Line plots are useful for ordering small sets of data. Histograms or bar graphs can be used to organize larger sets.

26. The second graph is misleading because the vertical scale is too small which makes small changes look large. Answers will vary.

APPENDIX C
Concepts in Statistics

Appendix C.1 Measures of Central Tendency and Dispersion

Vocabulary Check

1. measure, central tendency
2. modes, bimodal
3. variance, standard deviation
4. Quartiles

1. Mean $= \dfrac{5 + 12 + 7 + 14 + 8 + 9 + 7}{7} = \dfrac{62}{7} \approx 8.86$

Median: 8

Mode: 7

2. Mean $= \dfrac{30 + 37 + 32 + 39 + 33 + 34 + 32}{7} = \dfrac{237}{7} \approx 33.86$

Median: 33

Mode: 32

3. Mean $= \dfrac{5 + 12 + 7 + 24 + 8 + 9 + 7}{7} = \dfrac{72}{7} \approx 10.29$

Median: 8

Mode: 7

4. Mean $= \dfrac{20 + 37 + 32 + 39 + 33 + 34 + 32}{7} = \dfrac{227}{7} \approx 32.43$

Median: 33

Mode: 32

5. Mean $= \dfrac{5 + 12 + 7 + 14 + 9 + 7}{6} = \dfrac{54}{6} = 9$

Median: $\dfrac{7 + 9}{2} = 8$

Mode: 7

6. Mean $= \dfrac{30 + 37 + 32 + 39 + 34 + 32}{6} = \dfrac{204}{6} = 34$

Median: $\dfrac{32 + 34}{2} = 33$

Mode: 32

7. (a) The mean is sensitive to extreme values

(b) Mean: 14.86

Median: 14

Mode: 13

Each is increased by 6.

(c) Each will increase by k.

8. Mean $= \dfrac{805.69}{12} = \$67.14$

Median: $65.35

9. Mean $= \dfrac{410 + 260 + 320 + 320 + 460 + 150}{6} = \dfrac{1920}{6} = 320$

Median: 320

Mode: 320

10. Mean $= \dfrac{0(1) + 1(24) + 2(45) + 3(54) + 4(50) + 5(19) + 6(7)}{1 + 24 + 45 + 54 + 50 + 19 + 7} = \dfrac{613}{200} \approx 3.07$

Median: 3

Mode: 3

11. (a) Jay: $\dfrac{181 + 222 + 196}{3} = 199\frac{2}{3}$

Hank: $\dfrac{199 + 195 + 205}{3} = 199\frac{2}{3}$

Buck: $\dfrac{202 + 251 + 235}{3} = 229\frac{1}{3}$

(b) Adding all nine numbers, you obtain

$$\text{Mean} = \frac{1886}{9} = 209\frac{5}{9}.$$

(c) Median $= 202$ (four scores below 202 and four scores above 202)

12. (a) Mean $= \dfrac{5{,}510{,}000}{12} \approx \$459{,}167$

Mode: $500,000, $550,000, and $425,000

Median: $\dfrac{450{,}000 + 475{,}000}{2} = 462{,}500$

(b) Answers will vary.

13. There are many possible answers.
For example: $\{4, 4, 10\}$

14. There are many correct answers.
One possible set: $\{4, 4, 6, 7.5, 8.5\}$

15. The mean is 76.55 and the median is 82. The median is the best description.

16. Median and mode give most representative descriptions.

17. (a) Mean $= 12$, $\sigma \approx 2.83$

(b) Mean $= 20$, $\sigma \approx 2.83$

(c) Mean $= 12$, $\sigma \approx 1.41$

(d) Mean $= 9$, $\sigma \approx 1.41$

18. (a) $\bar{x} = 15$, $\sigma \approx 3.19$

(b) $\bar{x} = 15$, $\sigma \approx 2.83$

(c) $\bar{x} = 25$, $\sigma \approx 2.83$

(d) $\bar{x} = 5$, $\sigma \approx 3.19$

19. $\bar{x} = 6$, $v = 10$, $\sigma \approx 3.16$

20. $\bar{x} = 7$, $v = 22$, $\sigma \approx 4.69$

21. $\bar{x} = 2$, $v = \frac{4}{3}$, $\sigma \approx 1.15$

22. $\bar{x} = 2$, $v = 0$, $\sigma = 0$

23. $\bar{x} = 4$, $v = 4$, $\sigma \approx 2$

24. $\bar{x} = 3$, $v = 4$, $\sigma = 2$

25. $\bar{x} = 47$, $v = 226$, $\sigma \approx 15.03$

26. $\bar{x} = 1.1$, $v = 0.38$, $\sigma \approx 0.616$

27. $\bar{x} = 6$

$$\sigma = \sqrt{\frac{2^2 + 4^2 + 6^2 + 6^2 + 13^2 + 5^2}{6} - 6^2}$$
$$= \sqrt{\frac{286}{6} - 36}$$
$$= \sqrt{\frac{35}{3}} \approx 3.42$$

28. $\bar{x} = 300$

$$\sigma = \sqrt{\frac{246^2 + 336^2 + 473^2 + 167^2 + 219^2 + 359^2}{6} - 300^2}$$
$$= \sqrt{\frac{601{,}872}{6} - 300^2}$$
$$= \sqrt{10{,}312} \approx 101.55$$

29. $\bar{x} = 5.8$

$$\sigma = \sqrt{\frac{8.1^2 + 6.9^2 + 3.7^2 + 4.2^2 + 6.1^2}{5} - 5.8^2}$$
$$= \sqrt{2.712} \approx 1.65$$

30. $\bar{x} = 6.64$

$$\sigma = \sqrt{\frac{9^2 + 7.5^2 + 3.3^2 + 7.4^2 + 6^2}{5} - 6.64^2}$$
$$= \sqrt{\frac{238.9}{5} - 6.64^2}$$
$$= \sqrt{3.6904} \approx 1.92$$

31. $\bar{x} = 12$ and $|x_i - 12| = 8$ for all x_i. Hence, $\sigma = 8$.

32. All the numbers must be equal.

33. The mean will increase by 5. The standard deviation will not change.

34. Mean $\approx$ 362.46

Variance $\approx$ 2665.48

Standard deviation $\approx$ 51.63

100% of the data lies within two standard deviations of the mean.

[**Note:** Some graphing utilities give variance $\approx$ 2554.41 and standard deviation $\approx$ 50.54.]

35. $\bar{x} = 235$

$\sigma = 28$

$n = 600$

$1 - \frac{1}{2^2} = \frac{3}{4}$ lies within two standard deviations:

$[235 - 2(28), 235 + 2(28)] = [179, 291]$.

$1 - \frac{1}{3^2} = \frac{8}{9}$ lies within three standard deviations:

$[235 - 3(28), 235 + 3(28)] = [151, 319]$.

If $\sigma = 16$, then

$[235 - 2(16), 235 + 2(16)] = [203, 267]$

$[235 - 3(16), 235 + 3(16)] = [187, 283]$.

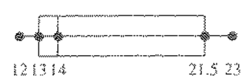
36. The first histogram has a smaller standard deviation.

37. (a) 12, 13, 13, 14, 14, 15, 20, 23, 23

Median: 14

Lower quartile is median of $\{12, 13, 13\} = 13$.

Upper quartile is median of $\{15, 20, 23, 23\} = 21.5$.

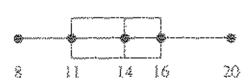
(b) 

38. (a) 8, 10, 11, 11, 11, 14, 14, 14, 16, 17, 20

Median: 14

Lower quartile is median of $\{8, 10, 11, 11, 11\} = 11$.

Upper quartile is median of $\{14, 14, 16, 17, 20\} = 16$.

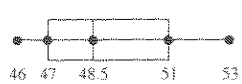
(b) 

39. (a) 46, 47, 47, 48, 48, 49, 50, 51, 52, 53

Median: $\frac{48 + 49}{2} = 48.5$

Lower quartile is median of $\{46, 47, 47, 48, 48\} = 47$.

Upper quartile is median of $\{49, 50, 51, 52, 53\} = 51$.

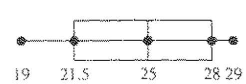
(b) 

40. (a) 19, 20, 21, 22, 24, 25, 25, 27, 28, 28, 28, 29

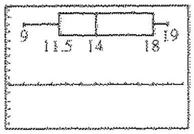
Median: 25

Lower quartile is median of $\{19, 20, 21, 22, 24, 25\} = 21.5$.

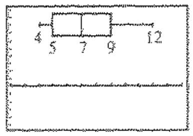
Upper quartile is median of $\{27, 28, 28, 28, 29\} = 28$.

(b) 

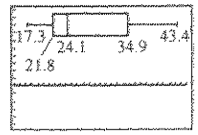
41.



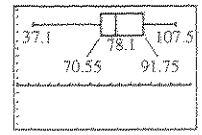
42.



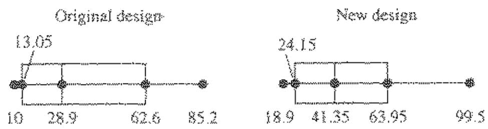
43.



44.



45.



From the plots, you can see that the lifetimes of the units in the new design are greater than the original design. The median increased by over 12 months.

Appendix C.2 Least Squares Regression

1.

	x	y	xy	x^2
	-4	1	-4	16
	-3	3	-9	9
	-2	4	-8	4
	-1	6	-6	1
Total	-10	14	-27	30

$n = 4$

$4b + (-10)a = 14$

$(-10)b + 30a = -27$

Solving this system, $a = 1.6$ and $b = 7.5$.

Answer: $y = 1.6x + 7.5$

2.

	x	y	xy	x^2
	0	-1	0	0
	2	0	0	4
	4	3	12	16
	6	5	30	36
Total	12	7	42	56

$n = 4$

$4b + 12a = 7$

$12b + 56a = 42$

Solving this system, $b = -1.4$ and $a = 1.05$.

Answer: $y = 1.05x - 1.4$

3.

	x	y	xy	x^2
	-3	1	-3	9
	-1	2	-2	1
	1	2	2	1
	4	3	12	16
Total	1	8	9	27

$n = 4$

$4b + a = 8$

$b + 27a = 9$

Solving this system, $a \approx 0.262$ and $b \approx 1.93$.

Answer: $y = 0.262x + 1.93$

4.

	x	y	xy	x^2
	0	-1	0	0
	2	1	2	4
	3	2	6	9
	5	3	15	25
Total	10	5	23	38

$n = 4$

$4b + 10a = 5$

$10b + 38a = 23$

Solving this system, $b \approx -0.769$ and $a \approx 0.808$.

Answer: $y = 0.808x - 0.769$

Appendix D Variation

Vocabulary Check

1. directly proportional
2. constant, variation
3. directly proportional
4. inverse
5. combined
6. jointly proportional

1. $y = kx$
$12 = k(5)$
$\frac{12}{5} = k$
$y = \frac{12}{5}x$

2. $y = kx$
$14 = k(2)$
$7 = k$
$y = 7x$

3. $y = kx$
$2050 = k(10)$
$205 = k$
$y = 205x$

4. $y = kx$
$580 = k(6)$
$\frac{290}{3} = k$
$y = \frac{290}{3}x$

5. $y = kx$
$33 = k(13)$
$\frac{33}{13} = k$
$y = \frac{33}{13}x$
When $x = 10$ inches, $y \approx 25.4$ centimeters.
When $x = 20$ inches, $y \approx 50.8$ centimeters.

6. $y = kx$
$53 = k(14)$
$\frac{53}{14} = k$
$y = \frac{53}{14}x$
5 gallons: $y = \frac{53}{14}(5) \approx 18.9$ liters
25 gallons: $y = \frac{53}{14}(25) \approx 94.6$ liters

7. $y = kx$
$5520 = k(150{,}000)$
$0.0368 = k$
$y = 0.0368x$
$y = 0.0368(200{,}000)$
$= \$7360$
The property tax is \$7360.

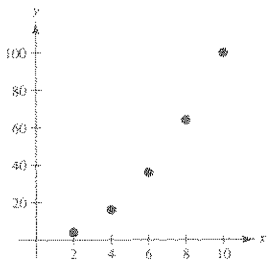
8. $y = kx$
$10.22 = k(145.99)$
$0.07 \approx k$
$y = 0.07x$
$y = 0.07(540.50)$
$y \approx 37.84$
The sales tax is \$37.84.

9. $d = kF$
$0.15 = k(265)$
$\frac{3}{5300} = k$
$d = \frac{3}{5300}F$
(a) $d = \frac{3}{5300}(90) \approx 0.05$ meter
(b) $0.1 = \frac{3}{5300}F$
$\frac{530}{3} = F$
$F = 176\frac{2}{3}$ newtons

10. $d = kF$
$0.12 = k(220)$
$\frac{3}{5500} = k$
$d = \frac{3}{5500}F$
$0.16 = \frac{3}{5500}F$
$\frac{880}{3} = F$
The required force is $293\frac{1}{3}$ newtons.

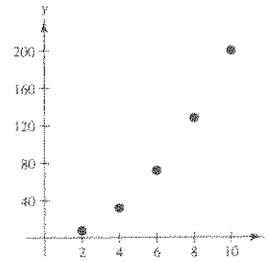
11. $k = 1$

x	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



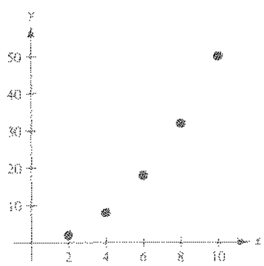
12. $k = 2$

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



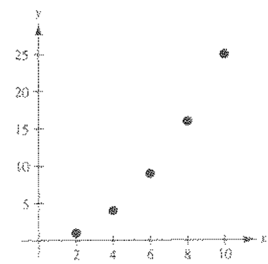
13. $k = \frac{1}{2}$

x	2	4	6	8	10
$y = kx^2$	2	8	18	32	50



14. $k = \frac{1}{4}$

x	2	4	6	8	10
$y = kx^2$	1	4	9	16	25



15.
$$d = kv^2$$
$$0.02 = k\left(\frac{1}{4}\right)^2$$
$$k = 0.32$$
$$d = 0.32v^2$$
$$0.12 = 0.32v^2$$
$$v^2 = \frac{0.12}{0.32} = \frac{3}{8}$$
$$v = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4} \approx 0.61 \text{ mi/hr}$$

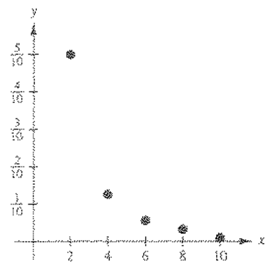
16. $d = kv^2$

If the velocity is doubled:
$$d = k(2v)^2$$
$$d = k \cdot 4v^2$$
$$\frac{4kv^2}{kv^2} = 4$$

d increases by a factor of 4 when velocity is doubled.

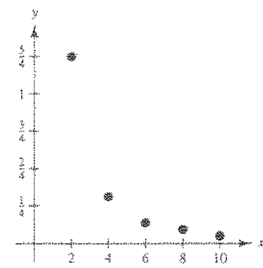
17. $k = 2$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{18}$	$\frac{1}{32}$	$\frac{1}{50}$



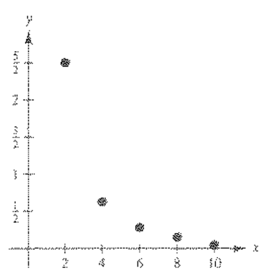
18. $k = 5$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{4}$	$\frac{5}{16}$	$\frac{5}{36}$	$\frac{5}{64}$	$\frac{1}{20}$



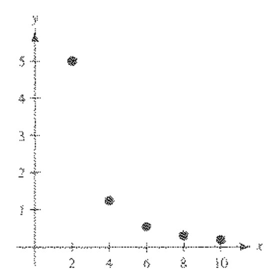
19. $k = 10$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	$\frac{5}{2}$	$\frac{5}{8}$	$\frac{5}{18}$	$\frac{5}{32}$	$\frac{1}{10}$



20. $k = 20$

x	2	4	6	8	10
$y = \frac{k}{x^2}$	5	$\frac{5}{4}$	$\frac{5}{9}$	$\frac{5}{16}$	$\frac{1}{5}$



21. The table represents the equation $y = 5/x$.

22. The table represents the equation $y = \frac{2}{5}x$.

23.
$$\begin{aligned} y &= kx \\ -7 &= k(10) \\ -\frac{7}{10} &= k \\ y &= -\frac{7}{10}x \end{aligned}$$

This equation checks with the other points given in the table.

24.
$$\begin{aligned} y &= \frac{k}{x} \\ 24 &= \frac{k}{5} \\ 120 &= k \end{aligned}$$

Thus, $y = 120/x$. This equation checks with the other points given in the table.

25. $A = kr^2$ **26.** $V = ke^3$ **27.** $y = \frac{k}{x^2}$ **28.** $h = \frac{k}{\sqrt{s}}$ **29.** $F = \frac{kg}{r^2}$

30. $z = kx^2y^3$ **31.** $P = \frac{k}{V}$ **32.** $R = kS(S - L)$ **33.** $R = k(T - T_e)$ **34.** $F = \frac{km_1m_2}{r^2}$

35. $A = \frac{1}{2}bh$

The area of a triangle is jointly proportional to its base and height.

36. $S = 4\pi r^2$

The surface area of a sphere varies directly as the square of the radius r.

37. $V = \frac{4}{3}\pi r^3$

The volume of a sphere varies directly as the cube of its radius.

38. $V = \pi r^2 h$

The volume of a right circular cylinder is jointly proportional to the height and the square of the radius.

39. $r = \dfrac{d}{t}$

Average speed is directly proportional to the distance and inversely proportional to the time.

40. $\omega = \sqrt{\dfrac{kg}{W}}$

ω varies directly as the square root of g and inversely as the square root of W.

(**Note:** The constant of proportionality is $\sqrt{k}$.)

41.
$$A = kr^2$$
$$9\pi = k(3)^2$$
$$\pi = k$$
$$A = \pi r^2$$

42.
$$y = \frac{k}{x}$$
$$3 = \frac{k}{25}$$
$$75 = k$$
$$y = \frac{75}{x}$$

43.
$$y = \frac{k}{x}$$
$$7 = \frac{k}{4}$$
$$28 = k$$
$$y = \frac{28}{x}$$

44.
$$z = kxy$$
$$64 = k(4)(8)$$
$$2 = k$$
$$z = 2xy$$

45.
$$F = krs^3$$
$$4158 = k(11)(3)^3$$
$$k = 14$$
$$F = 14rs^3$$

46.
$$P = \frac{kx}{y^2}$$
$$\frac{28}{3} = \frac{k(42)}{9^2}$$
$$\frac{28}{3} \cdot \frac{81}{42} = k$$
$$\frac{2 \cdot 27}{3} = k$$
$$18 = k$$
$$P = \frac{18x}{y^2}$$

47.
$$z = \frac{kx^2}{y}$$
$$6 = \frac{k(6)^2}{4}$$
$$\frac{24}{36} = k$$
$$\frac{2}{3} = k$$
$$z = \frac{2/3x^2}{y} = \frac{2x^2}{3y}$$

48.
$$v = \frac{kpq}{s^2}$$
$$1.5 = \frac{k(4.1)(6.3)}{(1.2)^2}$$
$$\frac{(1.5)(1.44)}{(4.1)(6.3)} = k$$
$$\frac{2.16}{25.83} = k$$
$$k = \frac{24}{287}$$
$$v = \frac{24pq}{287s^2}$$

49.

$$r = \frac{kl}{A},\ A = \pi r^2 = \frac{\pi d^2}{4}$$

$$r = \frac{4kl}{\pi d^2}$$

$$66.17 = \frac{4(1000)k}{\pi\left(\frac{0.0126}{12}\right)^2}$$

$$k \approx 5.73 \times 10^{-8}$$

$$r = \frac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$$

$$33.5 = \frac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$$

$$\frac{33.5\pi\left(\frac{0.0126}{12}\right)^2}{4(5.73 \times 10^{-8})} = l$$

$$l \approx 506 \text{ feet}$$

50. From Exercise 49:

$$k \approx 5.73 \times 10^{-8}$$

$$r = \frac{4(5.73 \times 10^{-8})l}{\pi d^2}$$

$$d = \sqrt{\frac{4(5.73 \times 10^{-8})l}{\pi r}}$$

$$d = \sqrt{\frac{4(5.73 \times 10^{-8})(14)}{\pi(0.05)}}$$

$$d \approx 0.0045 \text{ feet} = 0.054 \text{ inch}$$

51.

$$W = kmh$$

$$2116.8 = k(120)(1.8)$$

$$k = \frac{2116.8}{(120)(1.8)} = 9.8$$

$$W = 9.8mh$$

When $m = 100$ kilograms and $h = 1.5$ meters, we have $W = 9.8(100)(1.5) = 1470$ joules.

52.

$$P = kA = k(\pi r^2) = k\pi\left(\frac{d}{2}\right)^2$$

$$8.78 = k\pi\left(\frac{9}{2}\right)^2$$

$$\frac{4(8.78)}{81\pi} = k$$

$$k \approx 0.138$$

However, we do not obtain \$11.78 when $d = 12$ inches.

$$P = 0.138\pi\left(\frac{12}{2}\right)^2 \approx \$15.61$$

Instead, $k = \frac{11.78}{36\pi} \approx 0.104$.

For the 15-inch pizza, we have $k = \frac{4(14.18)}{225\pi} \approx 0.080$.

The price is not directly proportional to the surface area. The best buy is the 15-inch pizza.

53. $v = \frac{k}{A}$

$$v = \frac{k}{0.75A} = \frac{4}{3}\left(\frac{k}{A}\right)$$

The velocity is increased by one-third.

54. Load $= \dfrac{kwd^2}{l}$

(a) Load $= \dfrac{k(2w)d^2}{2l} = \dfrac{kwd^2}{l}$

The safe load is unchanged.

(b) Load $= \dfrac{k(2w)(2d)^2}{l} = \dfrac{8kwd^2}{l}$

The safe load is eight times as great.

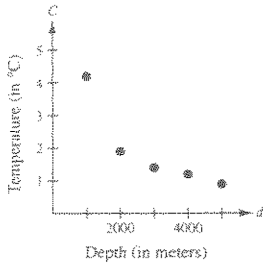
(c) Load $= \dfrac{k(2w)(2d)^2}{2l} = \dfrac{4kwd^2}{l}$

The safe load is four times as great.

(d) Load $= \dfrac{kw(d/2)^2}{l} = \dfrac{(1/4)kwd^2}{l}$

The safe load is one-fourth as great.

55. (a)



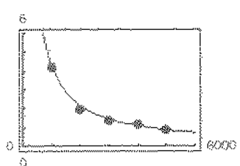
(b) Yes, the data appears to be modeled (approximately) by the inverse proportion model.

$4.2 = \dfrac{k_1}{1000}$ $\quad 1.9 = \dfrac{k_2}{2000}$ $\quad 1.4 = \dfrac{k_3}{3000}$ $\quad 1.2 = \dfrac{k_4}{4000}$ $\quad 0.9 = \dfrac{k_5}{5000}$

$4200 = k_1$ $\quad 3800 = k_2$ $\quad 4200 = k_3$ $\quad 4800 = k_4$ $\quad 4500 = k_5$

(c) Mean: $k = \dfrac{4200 + 3800 + 4200 + 4800 + 4500}{5} = 4300$, Model: $C = \dfrac{4300}{d}$

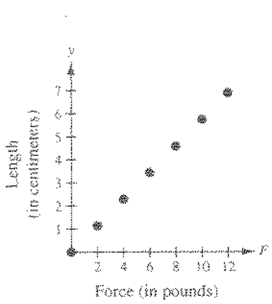
(d)



(e) $3 = \dfrac{4300}{d}$

$d = \dfrac{4300}{3} = 1433\frac{1}{3}$ meters

56. (a)



(b) It appears to fit Hooke's Law.

$k \approx \dfrac{6.9}{12} = 0.575$

(c) $y = kF$

$9 = 0.575F$

$F \approx 15.7$ pounds

57. False. y will increase if k is positive and y will decrease if k is negative.

58. False. E is jointly proportional (not "directly proportional") to the mass of an object and the square of its velocity.

59. The graph appears to represent $y = 4/x$, so y varies inversely as x.

60. The graph appears to represent $y = \frac{3}{2}x$ which is a direct variation.

APPENDIX E
Solving Linear Equations and Inequalities

Vocabulary Check

1. linear

2. equivalent inequalities

1. $x + 11 = 15$
$x = 15 - 11$
$x = 4$

2. $x + 3 = 9$
$x = 9 - 3$
$x = 6$

3. $x - 2 = 5$
$x = 5 + 2$
$x = 7$

4. $x - 5 = 1$
$x = 1 + 5$
$x = 6$

5. $3x = 12$
$x = \frac{12}{3}$
$x = 4$

6. $2x = 6$
$x = \frac{6}{2}$
$x = 3$

7. $\frac{x}{5} = 4$
$x = 4(5)$
$x = 20$

8. $\frac{x}{4} = 5$
$x = 5(4)$
$x = 20$

9. $8x + 7 = 39$
$8x = 32$
$x = 4$

10. $12x - 5 = 43$
$12x = 48$
$x = 4$

11. $24 - 7x = 3$
$-7x = -21$
$x = 3$

12. $13 + 6x = 61$
$6x = 48$
$x = 8$

13. $8x - 5 = 3x + 20$
$5x = 25$
$x = 5$

14. $7x + 3 = 3x - 17$
$4x = -20$
$x = -5$

15. $-2(x + 5) = 10$
$-2x - 10 = 10$
$-2x = 20$
$x = -10$

16. $4(3 - x) = 9$
$12 - 4x = 9$
$-4x = -3$
$x = \frac{3}{4}$

17. $2x + 3 = 2x - 2$
$3 = -2$
No solution

18. $8(x - 2) = 4(2x - 4)$
$8x - 16 = 8x - 16$
True for all real numbers.

19. $\frac{3}{2}(x + 5) - \frac{1}{4}(x + 24) = 0$
$\frac{3}{2}(x + 5) = \frac{1}{4}(x + 24)$
$12(x + 5) = 2(x + 24)$
$12x + 60 = 2x + 48$
$10x = -12$
$x = -\frac{12}{10}$
$x = -\frac{6}{5}$

20. $\frac{3}{2}x + \frac{1}{4}(x - 2) = 10$
$\frac{3}{2}x + \frac{1}{4}x - \frac{1}{2} = 10$
$\frac{7}{4}x = \frac{21}{2}$
$x = \frac{21}{2} \cdot \frac{4}{7}$
$x = 6$

21. $0.25x + 0.75(10 - x) = 3$

$25x + 75(10 - x) = 300$

$25x + 750 - 75x = 300$

$-50x = -450$

$x = 9$

22. $0.60x + 0.40(100 - x) = 50$

$0.60x + 40 - 0.40x = 50$

$0.20x = 10$

$x = \frac{10}{0.2}$

$= 50$

23. $x + 6 < 8$

$x < 8 - 6$

$x < 2$

24. $3 + x > -10$

$x > -10 - 3$

$x > -13$

25. $-x - 8 > -17$

$17 - 8 > x$

$9 > x$

$x < 9$

26. $-3 + x < 19$

$x < 19 + 3$

$x < 22$

27. $6 + x \le -8$

$x \le -8 - 6$

$x \le -14$

28. $x - 10 \ge -6$

$x \ge -6 + 10$

$x \ge 4$

29. $\frac{4}{5}x > 8$

$x > \frac{5}{4}(8)$

$x > 10$

30. $\frac{2}{3}x < -4$

$x < \frac{3}{2}(-4)$

$x < -6$

31. $-\frac{3}{4}x > -3$

$\frac{3}{4}x < 3$

$x < 4$

32. $-\frac{1}{6}x < -2$

$\frac{1}{6}x > 2$

$x > 12$

33. $4x < 12$

$x < 3$

34. $10x > -40$

$x > -\frac{40}{10}$

$x > -4$

35. $-11x \le -22$

$11x \ge 22$

$x \ge 2$

36. $-7x \ge 21$

$x \le \frac{21}{(-7)}$

$x \le -3$

37. $x - 3(x + 1) \ge 7$

$x - 3x - 3 \ge 7$

$-2x \ge 10$

$x \le -5$

38. $2(4x - 5) - 3x \le -15$

$8x - 10 - 3x \le -15$

$5x \le -5$

$x \le -1$

39. $7x - 12 < 4x + 6$

$3x < 18$

$x < 6$

40. $11 - 6x \le 2x + 7$

$4 \le 8x$

$\frac{1}{2} \le x$

$x \ge \frac{1}{2}$

41. $\frac{3}{4}x - 6 \le x - 7$

$1 \le \frac{1}{4}x$

$4 \le x$

$x \ge 4$

42. $3 + \frac{2}{7}x > x - 2$

$5 > \frac{5}{7}x$

$7 > x$

$x < 7$

43. $3.6x + 11 \ge -3.4$

$3.6x \ge -14.4$

$x \ge \frac{-14.4}{3.6}$

$x \ge -4$

44. $15.6 - 1.3x < -5.2$

$20.8 < 1.3x$

$16 < x$

$x > 16$

APPENDIX F
Systems of Inequalities

Appendix F.1 Solving Systems of Inequalities

Vocabulary Check

1. solution
2. graph
3. linear
4. point, equilibrium

1. $x < 2$

   Vertical boundary; Matches graph (g).

2. $y \geq 3$

   Region above or on horizontal line $y = 3$; Matches (d).

3. $2x + 3y \geq 6$

   $y \geq -\frac{2}{3}x + 2$

   Line with negative slope; Matches (a).

4. $2x - y \leq -2 \implies y \geq 2x + 2$

   Region above or on line $y = 2x + 2$; Matches (h).

5. $x^2 + y^2 < 9$

   Circular boundary; Matches (e).

6. $(x - 2)^2 + (y - 3)^2 > 9$

   Region outside circle; Matches (b).

7. $xy > 1$ or $y > \dfrac{1}{x}$

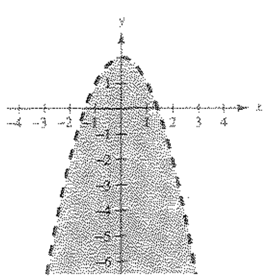
   Matches (f).

8. $y \leq 1 - x^2$

   Region below or on parabola; Matches (c).

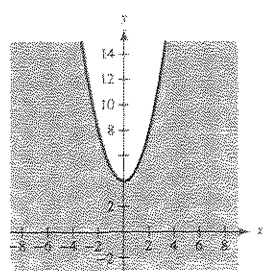
9. $y < 2 - x^2$

   Graph the parabola $y = 2 - x^2$. The region lies below the parabola.

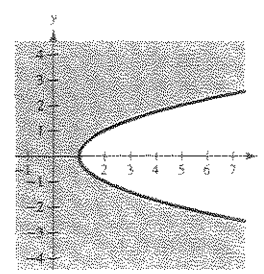


10. $y - 4 \leq x^2$

    $y \leq 4 + x^2$

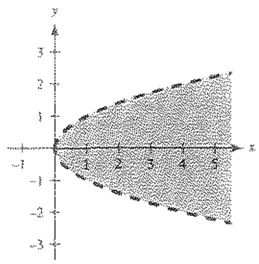


11. $y^2 + 1 \geq x$



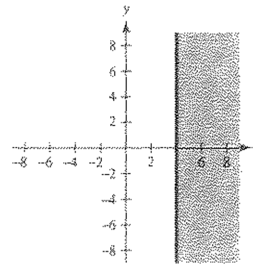
12. $y^2 - x < 0$

$x > y^2$

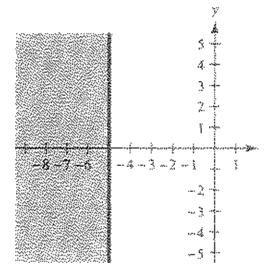


13. $x \ge 4$

Using a solid line, graph the vertical line $x = 4$ and shade to the right of this line.

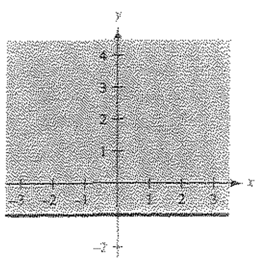


14. $x \le -5$



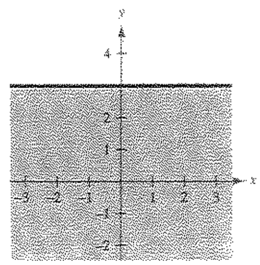
15. $y \ge -1$

Using a solid line, graph the horizontal line $y = -1$ and shade above this line.



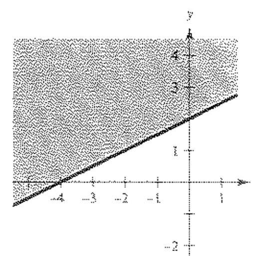
16. $y \le 3$

Using a solid line, graph the horizontal line $y = 3$, and shade below this line.



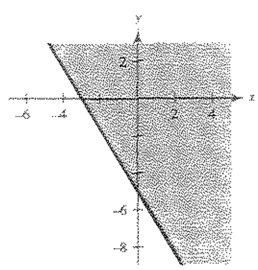
17. $2y - x \ge 4$

Using a solid line, graph $2y - x = 4$, and then shade above the line. (Use $(0, 0)$ as a test point.)

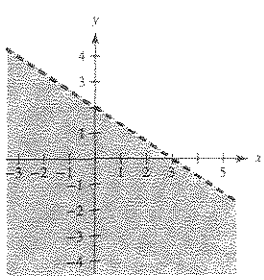


18. $5x + 3y \ge -15$

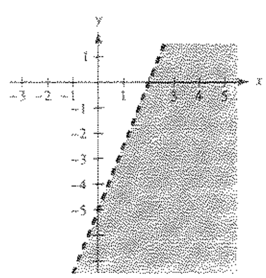
Using a solid line, graph $5x + 3y = -15$, and shade above the line. (Use $(0, 0)$ as a test point.)



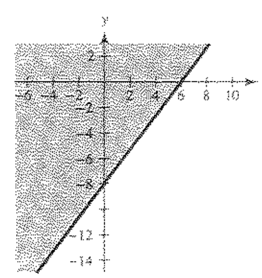
19. $2x + 3y < 6$



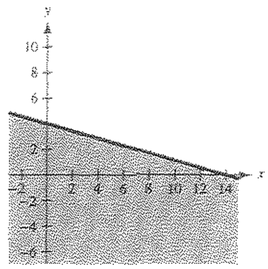
20. $5x - 2y > 10$



21. $4x - 3y \le 24$

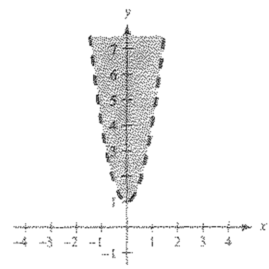


22. $2x + 7y \leq 28$

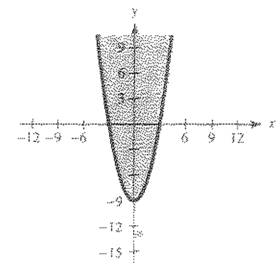


23. $y > 3x^2 + 1$

Sketch the parabola $y = 3x^2 + 1$. The region lies above the parabola.

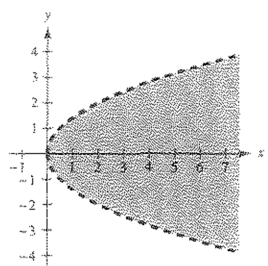


24. $y + 9 \geq x^2$



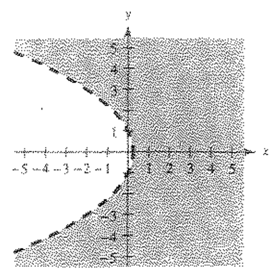
25. $2x - y^2 > 0$

$2x > y^2$

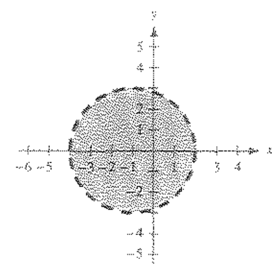


26. $4x + y^2 > 1$

Region to right of parabola

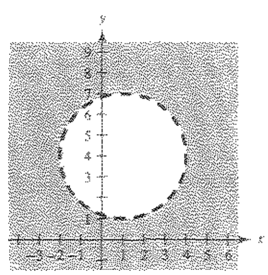


27. $(x + 1)^2 + y^2 < 9$

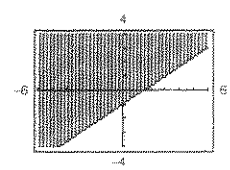


28. $(x - 1)^2 + (y - 4)^2 > 9$

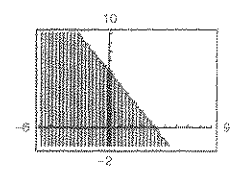
Region outside of circle



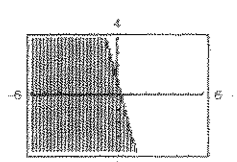
29. $y \geq \frac{2}{3}x - 1$



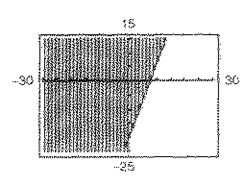
30. $y \leq 6 - \frac{3}{2}x$



31. $y < -3.8x + 1.1$

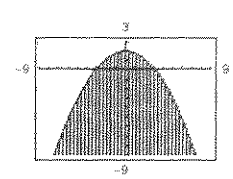


32. $y \geq -20.74 + 2.66x$



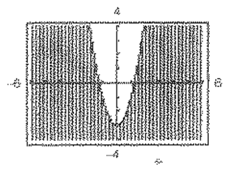
33. $x^2 + 5y - 10 \leq 0$

$$y \leq 2 - \frac{x^2}{5}$$

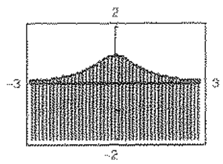


34. $2x^2 - y - 3 > 0$

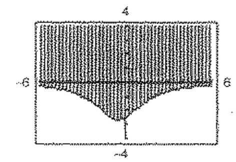
$y < 2x^2 - 3$



35. $y \le \dfrac{1}{1 + x^2}$

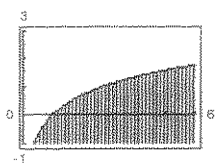


36. $y > \dfrac{-10}{x^2 + x + 4}$

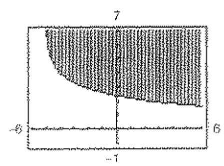


37. $y < \ln x$

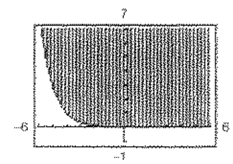
Using a dashed line, graph $y = \ln x$, and shade to the right of the curve. (Use $(2, 0)$ as a test point.)



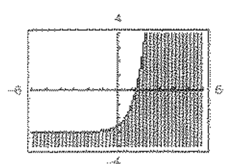
38. $y \ge 4 - \ln(x + 5)$



39. $y > 3^{-x-4}$



40. $y \le 2^{2x-1} - 3$



41. The line through $(0, 2)$ and $(3, 0)$ is $y = -\frac{2}{3}x + 2$. For the shaded region above the line, we have:

$$y > -\frac{2}{3}x + 2$$
$$3y > -2x + 6$$
$$2x + 3y > 6$$
$$\frac{x}{3} + \frac{y}{2} > 1$$

42. The parabola through $(-2, 0)$, $(0, -4)$, $(2, 0)$ is $y = x^2 - 4$. For the shaded region inside the parabola, we have $y \ge x^2 - 4$.

43. The circle shown is $x^2 + y^2 = 9$. For the shaded region inside the circle, we have $x^2 + y^2 \le 9$.

44. The region to the right of the vertical line $x = 5$. Thus, $x > 5$.

45. (a) $(0, 2)$ is a solution: $-2(0) + 5(2) \ge 3$

$$2 < 4$$
$$-4(0) + 2(2) < 7$$

(b) $(-6, 4)$ is not a solution: $4 \nless 4$

(c) $(-8, -2)$ is not a solution: $-4(-8) + 2(-2) \nless 7$

(d) $(-3, 2)$ is not a solution: $-4(-3) + 2(2) \nless 7$

46. $x^2 + y^2 \ge 36$

$-3x + y \le 10$

$\frac{2}{3}x - y \ge 5$

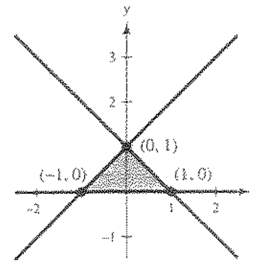
(a) $(-1, 7)$ is not a solution because $(-1)^2 + 7^2 = 50 < 36$.

(b) $(-5, 1)$ is not a solution because $-3(-5) + 1 = 16 > 10$.

(c) $(6, 0)$ is not a solution because $\frac{2}{3}(6) - 0 = 4 < 5$.

(d) $(4, -8)$ is a solution.

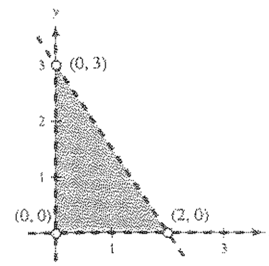
47. $\begin{cases} x + y \le 1 \\ -x + y \le 1 \\ y \ge 0 \end{cases}$



First, find the points of intersection of each pair of equations.

Vertex A	*Vertex B*	*Vertex C*
$\begin{cases} x + y = 1 \\ -x + y = 1 \end{cases}$	$\begin{cases} x + y = 1 \\ y = 0 \end{cases}$	$\begin{cases} -x + y = 1 \\ y = 0 \end{cases}$
$(0, 1)$	$(1, 0)$	$(-1, 0)$

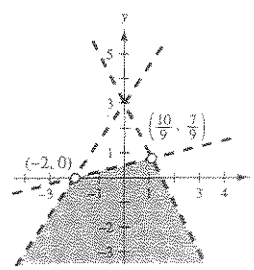
48. $\begin{cases} 3x + 2y < 6 \\ x < 0 \\ y < 0 \end{cases}$



First, find the points of intersection of each pair of equations.

Vertex A	*Vertex B*	*Vertex C*
$\begin{cases} 3x + 2y = 6 \\ x = 0 \end{cases}$	$\begin{cases} x = 0 \\ y = 0 \end{cases}$	$\begin{cases} 3x + 2y = 6 \\ y = 0 \end{cases}$
$(0, 3)$	$(0, 0)$	$(2, 0)$

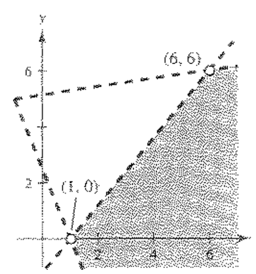
49. $\begin{cases} -3x + 2y < 6 \\ x - 4y > -2 \\ 2x + y < 3 \end{cases}$



First, find the points of intersection of each pair of equations.

Vertex A	*Vertex B*	*Vertex C*
$\begin{cases} -3x + 2y = 6 \\ x - 4y = -2 \end{cases}$	$\begin{cases} -3x + 2y = 6 \\ 2x + y = 3 \end{cases}$	$\begin{cases} x - 4y = -2 \\ 2x + y = 3 \end{cases}$
$(-2, 0)$	$(0, 3)$	$\left(\frac{10}{9}, \frac{7}{9}\right)$

50. $\begin{cases} x - 7y > -36 \\ 5x + 2y > 5 \\ 6x - 5y > 6 \end{cases}$



First, find the points of intersection of each pair of equations.

Vertex A	*Vertex B*	*Vertex C*
$\begin{cases} x - 7y = -36 \\ 5x + 2y = 5 \end{cases}$	$\begin{cases} 5x + 2y = 5 \\ 6x - 5y = 6 \end{cases}$	$\begin{cases} x - 7y = -36 \\ 6x - 5y = 6 \end{cases}$
$(-1, 5)$	$(1, 0)$	$(6, 6)$

51. $3x + y \le y^2$

$x - y > 0$

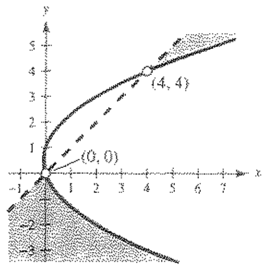
The curves given by $3x + y = y^2$ and $x - y = 0$ intersect as follows:

$$3x + x = x^2$$

$$4x = x^2$$

$$x = 0, 4$$

Intersection points:
$(0, 0), (4, 4)$



52. $y^2 - 3x \ge 9 \implies x \le \dfrac{y^2}{3} - 3$

$x + y \ge -3 \implies x \ge -y - 3$

The curves intersect where

$$\frac{y^2}{3} - 3 = -y - 3$$

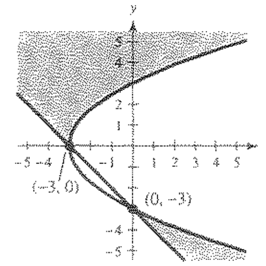
$$y^2 + 3y = 0$$

$$y(y + 3) = 0$$

$$y = 0, -3.$$

Intersection points:

$(-3, 0), (0, -3)$



53. $2x + y < 2 \implies y < 2 - 2x$

$x + 3y > 2 \implies y > \frac{1}{3}(2 - x)$

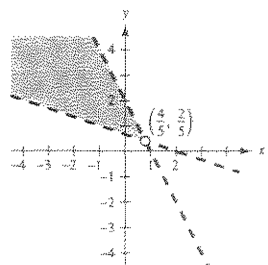
$2 - 2x = \frac{1}{3}(2 - x)$

$6 - 6x = (2 - x)$

$4 = 5x$

$x = \frac{4}{5}$

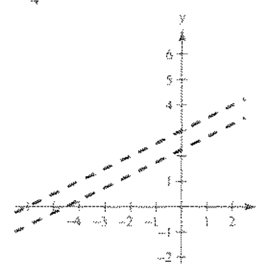
Intersection: $\left(\frac{4}{5}, \frac{2}{5}\right)$



54. $x - 2y-6 \implies y > \frac{1}{2}x + 3$

$2x - 4y-9 \implies y < \frac{1}{2}x + \frac{9}{4}$

No solution



55. $\begin{cases} x < y^2 \\ x > y + 2 \end{cases}$

Points of intersection:

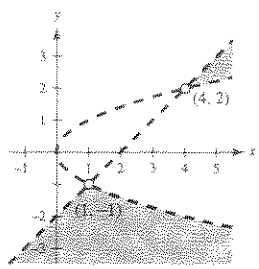
$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y + 1)(y - 2) = 0$$

$$y = -1, 2$$

$(1, -1), (4, 2)$



56. $x - y^2 > 0$

$x - y < 2$

Points of intersection:

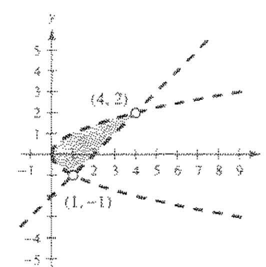
$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y + 1)(y - 2) = 0$$

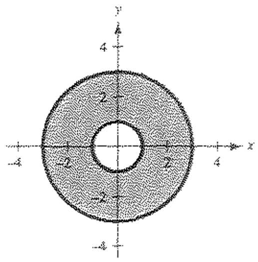
$$y = -1, 2$$

$(1, -1), (4, 2)$

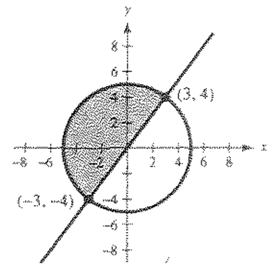


57. $\begin{cases} x^2 + y^2 \le 9 \\ x^2 + y^2 \ge 1 \end{cases}$

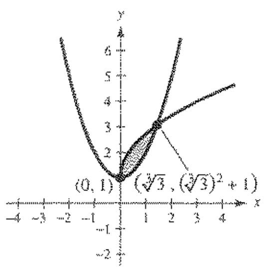
There are no points of intersection. The region in common to both inequalities is the region between the circles.



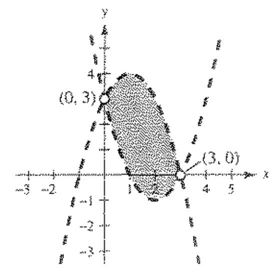
58. $\begin{cases} x^2 + y^2 \le 25 & \text{Circle and interior} \\ 4x - 3y \le 0 & \text{Above line} \end{cases}$



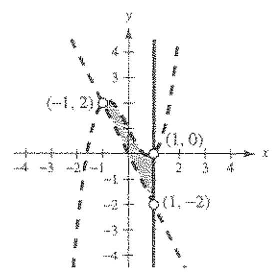
59. $\begin{cases} y \le \sqrt{3x} + 1 \\ y \ge x^2 + 1 \end{cases}$



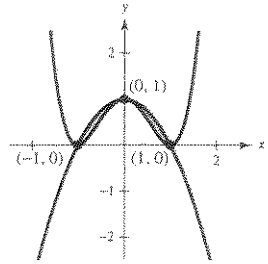
60. $\begin{cases} y < -x^2 + 2x + 3 \\ y > x^2 - 4x + 3 \end{cases}$



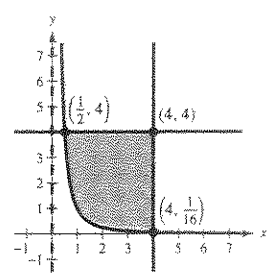
61. $\begin{cases} y < x^3 - 2x + 1 \\ y > -2x \\ x \le 1 \end{cases}$



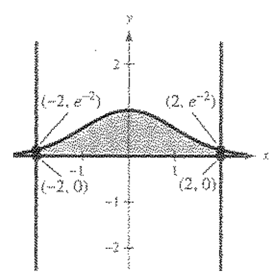
62. $\begin{cases} y \ge x^4 - 2x^2 + 1 \\ y \le 1 - x^2 \end{cases}$



63. $\begin{cases} x^2y \ge 1 \\ 0 < x \le 4 \\ y \le 4 \end{cases}$



64. $\begin{cases} y \le e^{-x^2/2} \\ y \ge 0 \\ -2 \le x \le 2 \end{cases}$



65. $y < -x + 4 \implies \dfrac{x}{4} + \dfrac{y}{4} < 1$

$x \ge 0 \qquad\qquad x \ge 0$

$y \ge 0 \qquad\qquad y \ge 0$

66. (0, 6), (3, 0)

Line: $y \le 6 - 2x$

(0, −3), (3, 0)

Line: $y \ge x - 3$

$x \ge 1$

67. (0, 4), (4, 0)

Line: $y \le 4 - x$

(0, 2), (8, 0)

Line: $y \le -\dfrac{1}{4}x + 2$

$x \ge 0, \ y \ge 0$

68. The lines have equations $y = -\frac{1}{3}x + 2$ and $y = 4 - x$. They intersect at $(3, 1)$.

$x + 3y < 6$

$x + y < 4$

$x \geq 0$

$y \geq 0$

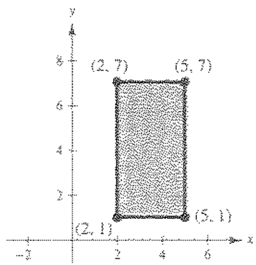
69. Circle of radius 2 and center $(0, 2)$

$x^2 + (y - 2)^2 \leq 4$

70. Circle: $x^2 + y^2 > 4$

71. $\begin{cases} x \geq 2 \\ x \leq 5 \\ y \geq 1 \\ y \leq 7 \end{cases}$

Thus, $2 \leq x \leq 5, 1 \leq y \leq 7.$



72. Parallelogram with vertices at $(0, 0), (4, 0), (1, 4), (5, 4)$

$(0, 0), (4, 0)$: $y \geq 0$

$(4, 0), (5, 4)$: $4x - y \leq 16$

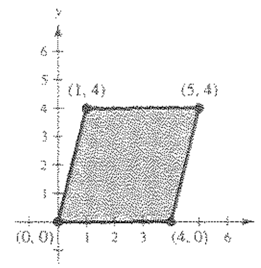
$(1, 4), (5, 4)$: $y \leq 4$

$(0, 0), (1, 4)$: $4x - y \geq 0$

$4x - y \geq 0$

$4x - y \leq 16$

$0 \leq y \leq 4$



73. $(0, 0), (5, 0)$

Line: $y \geq 0$

$(0, 0), (2, 3)$

Line: $y \leq \frac{3}{2}x$

$(2, 3), (5, 0)$

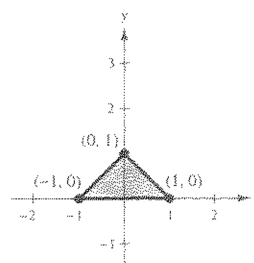
Line: $y \leq -x + 5$

74. Triangle with vertices at $(-1, 0), (1, 0), (0, 1)$

$(-1, 0), (1, 0)$: $y \geq 0$

$(-1, 0), (0, 1)$: $y \leq x + 1$

$(0, 1), (1, 0)$: $y \leq -x + 1$



75. Demand = Supply

$50 - 0.5x = 0.125x$

$50 = 0.625x$

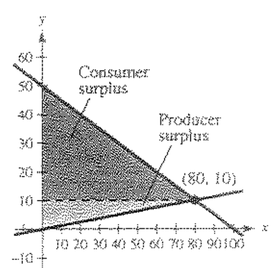
$x = 80$

$p = 10$

Point of equilibrium: $(80, 10)$

Consumer surplus $= \frac{1}{2}(40)(80) = 1600$

Producer surplus $= \frac{1}{2}(10)(80) = 400$



76. Demand = Supply

$100 - 0.05x = 25 + 0.1x$

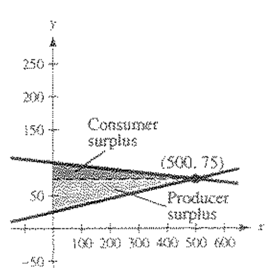
$75 = 0.15x$

$x = 500$

$p = 75$

Consumer surplus $= \frac{1}{2}(500)(25) = 6250$

Producer surplus $= \frac{1}{2}(500)(50) = 12{,}500$



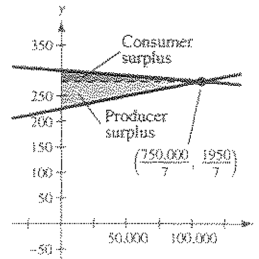
77. Demand = Supply

$$300 - 0.0002x = 225 + 0.0005x$$
$$75 = 0.0007x$$
$$x = \frac{75}{0.0007} = \frac{750{,}000}{7}$$

Equilibrium point: $\left(\frac{750{,}000}{7}, \frac{1950}{7}\right) \approx (107{,}142.86, 278.57)$

Consumer surplus: $\frac{(107{,}142.86)(300 - 278.57)}{2} \approx 1{,}148{,}036$

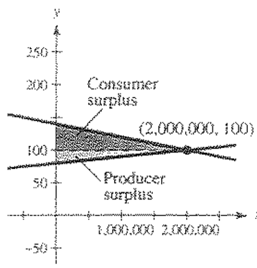
Producer surplus: $\frac{(107{,}142.86)(278.57 - 225)}{2} \approx 2{,}869{,}822$



78. Demand = Supply

$$140 - 0.00002x = 80 + 0.00001x$$
$$60 = 0.00003x$$
$$2{,}000{,}000 = x$$
$$100 = p$$

Point of equilibrium: $(2{,}000{,}000, 100)$



The consumer surplus is the area of the triangle bounded by

$$\begin{cases} p \le 140 - 0.00002x \\ p \ge 100 \\ x \ge 0. \end{cases}$$

$$\begin{aligned} \text{Consumer surplus} &= \tfrac{1}{2}(\text{base})(\text{height}) \\ &= \tfrac{1}{2}(2{,}000{,}000)(40) \\ &= 40{,}000{,}000 \text{ or } \$40 \text{ million} \end{aligned}$$

The producer surplus is the area of the triangle bounded by

$$\begin{cases} p \ge 80 + 0.00001x \\ p \le 100 \\ x \ge 0. \end{cases}$$

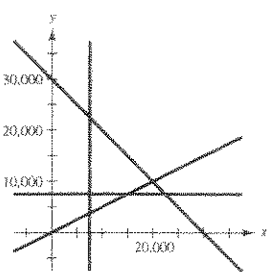
$$\begin{aligned} \text{Producer surplus} &= \tfrac{1}{2}(\text{base})(\text{height}) \\ &= \tfrac{1}{2}(2{,}000{,}000)(20) \\ &= 20{,}000{,}000 \text{ or } \$20 \text{ million} \end{aligned}$$

79. $x + y \le 30{,}000$

$x \ge 7500$

$y \ge 7500$

$x \ge 2y$



80. Let x be the number of \$20 tickets.

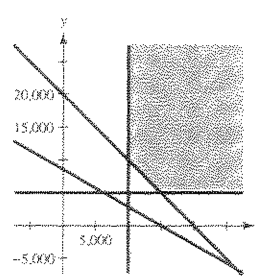
Let y be the number of \$35 tickets.

$x + y \ge 20{,}000$

$x \ge 10{,}000$

$y \ge 5{,}000$

$20x + 35y \ge 300{,}000$



81. (a) Let x = number of ounces of food X.

Let y = number of ounces of food Y.

Calcium: $20x + 10y \geq 280$

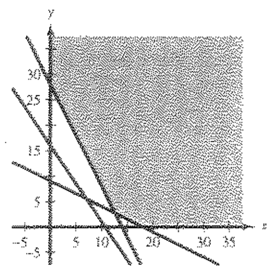
Iron: $15x + 10y \geq 160$

Vitamin B: $10x + 20y \geq 180$

$x \geq 0$

$y \geq 0$

(b)



82. x = number of model A

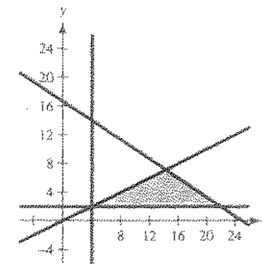
y = number of model B

Demand: $x \geq 2y$

Cost: $8x + 12y \leq 200$

Inventory: $x \geq 4$

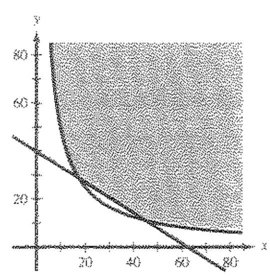
$y \geq 2$



83. (a)

$xy \geq 500$	Body-building space
$2x + \pi y \geq 125$	Track (two semi-circles and two lengths)
$x \geq 0$	Physical constraint
$y \geq 0$	Physical constraint

(b)



84. Let x = radius of smaller circle.

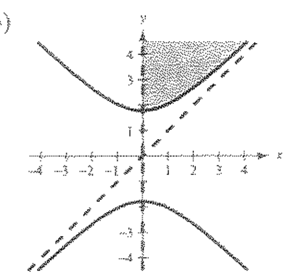
Let y = radius of larger circle.

(a) Constraints on circles: $\pi y^2 - \pi x^2 \geq 10$

$x > 0$

$y > x$

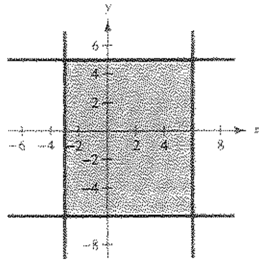
(b)



(c) The line is an asymptote to the boundary. The larger the circles, the closer the radii can be and the constraint still be satisfied.

85. Area = $9 \cdot 11 = 99$ square units

True



86. False

$3x + y^2 \geq 2$ is outside the parabola $3x = 2 - y^2$.

87. Test a point on either side of the boundary.

88. Answers will vary.

Appendix F.2 Linear Programming

Vocabulary Check

1. optimization **2.** objective function **3.** constraints, feasible solutions

1. $z = 3x + 5y$

At $(0, 6)$: $z = 3(0) + 5(6) = 30$

At $(0, 0)$: $z = 3(0) + 5(0) = 0$

At $(6, 0)$: $z = 3(6) + 5(0) = 18$

The minimum value is 0 at $(0, 0)$.

The maximum value is 30 at $(0, 6)$.

2. $z = 2x + 8y$

At $(0, 4)$: $z = 2(0) + 8(4) = 32$

At $(0, 0)$: $z = 2(0) + 8(0) = 0$

At $(2, 0)$: $z = 2(2) + 8(0) = 4$

The maximum value is 32 at $(0, 4)$.

The minimum value is 0 at $(0, 0)$.

3. $z = 10x + 7y$

At $(0, 6)$: $z = 10(0) + 7(6) = 42$

At $(0, 0)$: $z = 10(0) + 7(0) = 0$

At $(6, 0)$: $z = 10(6) + 7(0) = 60$

The minimum value is 0 at $(0, 0)$.

The maximum value is 60 at $(6, 0)$.

4. $z = 7x + 3y$

At $(0, 4)$: $z = 7(0) + 3(4) = 12$

At $(0, 0)$: $z = 7(0) + 3(0) = 0$

At $(2, 0)$: $z = 7(2) + 3(0) = 14$

The maximum value is 14 at $(2, 0)$.

The minimum value is 0 at $(0, 0)$.

5. $z = 3x + 2y$

$x + 3y = 15 \implies y = \frac{1}{3}(15 - x)$

$4x + y = 16 \implies y = (16 - 4x)$

$$\begin{aligned} \tfrac{1}{3}(15 - x) &= 16 - 4x \\ (15 - x) &= 48 - 12x \\ 11x &= 33 \\ x &= 3 \\ y &= 4 \end{aligned}$$

At $(0, 0)$: $z = 0$

At $(0, 5)$: $z = 10$

At $(4, 0)$: $z = 12$

At $(3, 4)$: $z = 17$

The minimum value is 0 at $(0, 0)$.

The maximum value is 17 at $(3, 4)$.

6. $z = 4x + 3y$

At $(0, 4)$: $z = 4(0) + 3(4) = 12$

At $(3, 0)$: $z = 4(3) + 3(0) = 12$

At $(5, 3)$: $z = 4(5) + 3(3) = 29$

At $(0, 2)$: $z = 4(0) + 3(2) = 6$

The maximum value is 29 at $(5, 3)$.

The minimum value is 6 at $(0, 2)$.

7. $z = 5x + 0.5y$

At $(0, 0)$: $z = 0$

At $(0, 5)$: $z = 2.5$

At $(4, 0)$: $z = 20$

At $(3, 4)$: $z = 17$

The minimum value is 0 at $(0, 0)$.

The maximum value is 20 at $(4, 0)$.

8. $z = x + 6y$

At $(0, 4)$: $z = 0 + 6(4) = 24$

At $(3, 0)$: $z = 3 + 6(0) = 3$

At $(5, 3)$: $z = 5 + 6(3) = 23$

At $(0, 2)$: $z = 0 + 6(2) = 12$

The maximum value is 24 at $(0, 4)$.

The minimum value is 3 at $(3, 0)$.

9. $z = 10x + 7y$

At $(0, 45)$: $z = 10(0) + 7(45) = 315$

At $(30, 45)$: $z = 10(30) + 7(45) = 615$

At $(60, 20)$: $z = 10(60) + 7(20) = 740$

At $(60, 0)$: $z = 10(60) + 7(0) = 600$

At $(0, 0)$: $z = 10(0) + 7(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 740 at $(60, 20)$.

10. $z = 50x + 35y$

At $(0, 800)$: $z = 50(0) + 35(800) = 28{,}000$

At $(900, 0)$: $z = 50(900) + 35(0) = 45{,}000$

At $(675, 0)$: $z = 50(675) + 35(0) = 33{,}750$

At $(0, 600)$: $z = 50(0) + 35(600) = 21{,}000$

The maximum value is 45,000 at $(900, 0)$.

The minimum value is 21,000 at $(0, 600)$.

11. $z = 25x + 30y$

At $(0, 45)$: $z = 25(0) + 30(45) = 1350$

At $(30, 45)$: $z = 25(30) + 30(45) = 2100$

At $(60, 20)$: $z = 25(60) + 30(20) = 2100$

At $(60, 0)$: $z = 25(60) + 30(0) = 1500$

At $(0, 0)$: $z = 25(0) + 30(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 2100 at any point along the line segment connecting $(30, 45)$ and $(60, 20)$.

12. $z = 15x + 20y$

At $(0, 800)$: $z = 16{,}000$

At $(900, 0)$: $z = 13{,}500$

At $(675, 0)$: $z = 10{,}125$

At $(0, 600)$: $z = 12{,}000$

The minimum value is 10,125 at $(675, 0)$.

The maximum value is 16,000 at $(0, 800)$.

13. $z = 6x + 10y$

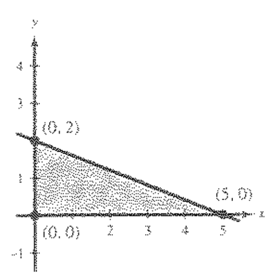
At $(0, 2)$: $z = 6(0) + 10(2) = 20$

At $(5, 0)$: $z = 6(5) + 10(0) = 30$

At $(0, 0)$: $z = 6(0) + 10(0) = 0$

The minimum value is 0 at $(0, 0)$.

The maximum value is 30 at $(5, 0)$.



14.
$x \ge 0$

$y \ge 0$

$x + \frac{1}{2}y \le 4$

$z = 7x + 8y$

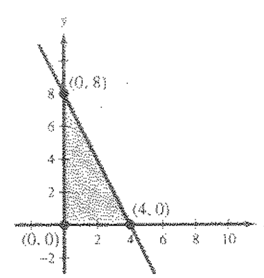
At $(0, 0)$: $z = 0$

At $(4, 0)$: $z = 28$

At $(0, 8)$: $z = 64$

The maximum value is 64 at $(0, 8)$.

The minimum value is 0 at $(0, 0)$.



15. $z = 3x + 4y$

At $(0, 0)$: $z = 0$

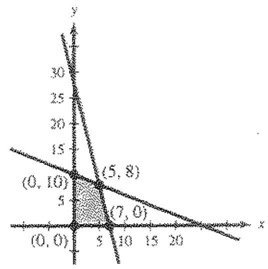
At $(7, 0)$: $z = 21$

At $(0, 10)$: $z = 40$

At $(5, 8)$: $z = 47$

The minimum value is 0 at $(0, 0)$.

The maximum value is 47 at $(5, 8)$.



16. $z = 4x + 5y$

At $(0, 0)$: $z = 4(0) + 5(0) = 0$

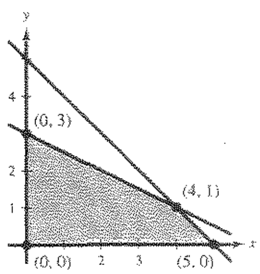
At $(5, 0)$: $z = 4(5) + 5(0) = 20$

At $(4, 1)$: $z = 4(4) + 5(1) = 21$

At $(0, 3)$: $z = 4(0) + 5(3) = 15$

The maximum value is 21 at $(4, 1)$.

The minimum value is 0 at $(0, 0)$.



17. $z = x + 2y$

At $(0, 0)$: $z = 0 + 2(0) = 0$

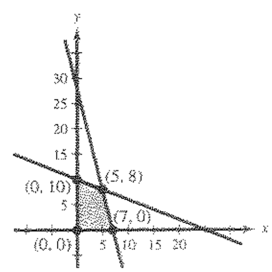
At $(0, 10)$: $z = 0 + 2(10) = 20$

At $(5, 8)$: $z = 5 + 2(8) = 21$

At $(7, 0)$: $z = 7 + 2(0) = 7$

The minimum value is 0 at $(0, 0)$.

The maximum value is 21 at $(5, 8)$.



18. $z = 2x + 4y$

At $(0, 0)$: $z = 2(0) + 4(0) = 0$

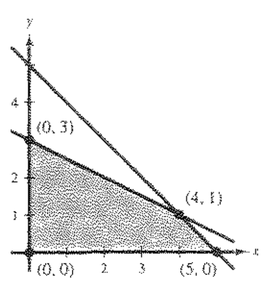
At $(0, 3)$: $z = 2(0) + 4(3) = 12$

At $(4, 1)$: $z = 2(4) + 4(1) = 12$

At $(5, 0)$: $z = 2(5) + 4(0) = 10$

The minimum value is 0 at $(0, 0)$.

The maximum value is 12 at any point on the line segment joining $(0, 3)$ and $(4, 1)$.



19. $z = 2x$

At $(0, 0)$: $z = 2(0) = 0$

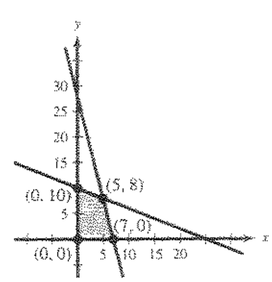
At $(0, 10)$: $z = 2(0) = 0$

At $(5, 8)$: $z = 2(5) = 10$

At $(7, 0)$: $z = 2(7) = 14$

The maximum value is 14 at $(7, 0)$.

The minimum value is 0 along the line segment joining $(0, 0)$ and $(0, 10)$.



20. $z = 3y$

At $(0, 0)$: $z = 3(0) = 0$

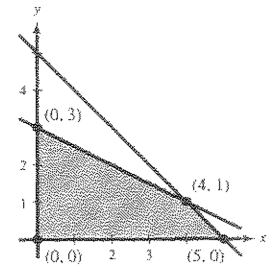
At $(0, 3)$: $z = 3(3) = 9$

At $(4, 1)$: $z = 3(1) = 3$

At $(5, 0)$: $z = 3(0) = 0$

The maximum value is 9 at $(0, 3)$.

The minimum value is 0 at any point on the line segment joining $(0, 0)$ and $(5, 0)$.



21. $z = 4x + y$

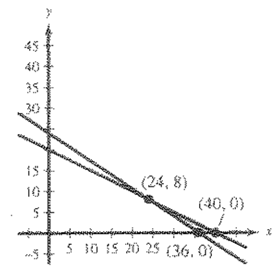
At $(36, 0)$: $z = 4(36) + 0 = 144$

At $(40, 0)$: $z = 4(40) + 0 = 160$

At $(24, 8)$: $z = 4(24) + 8 = 104$

The minimum value is 104 at $(24, 8)$.

The maximum value is 160 at $(40, 0)$.



22. $z = x$

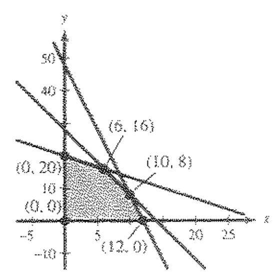
At $(0, 0)$: $z = 0$

At $(12, 0)$: $z = 12$

At $(10, 8)$: $z = 10$

At $(6, 16)$: $z = 6$

At $(0, 20)$: $z = 0$



The maximum value is 12 at $(12, 0)$. The minimum value is 0 at any point along the line segment connecting $(0, 0)$ and $(0, 20)$.

23. $z = x + 4y$

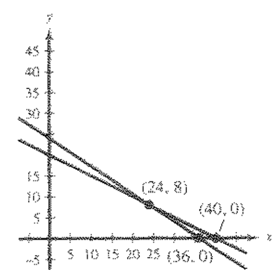
At $(36, 0)$: $z = 36 + 4(0) = 36$

At $(40, 0)$: $z = 40 + 4(0) = 40$

At $(24, 8)$: $z = 24 + 4(8) = 56$

The minimum value is 36 at $(36, 0)$.

The maximum value is 56 at $(24, 8)$.



24. $z = y$

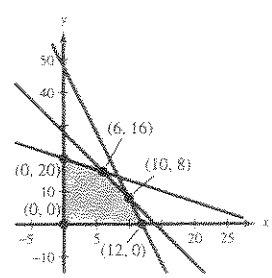
At $(0, 0)$: $z = 0$

At $(12, 0)$: $z = 0$

At $(10, 8)$: $z = 8$

At $(6, 16)$: $z = 16$

At $(0, 20)$: $z = 20$



The maximum value is 20 at $(0, 20)$.

The minimum value is 0 at any point along the line segment connecting $(0, 0)$ and $(12, 0)$.

25. $z = 2x + 3y$

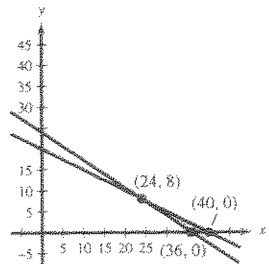
At $(36, 0)$: $z = 2(36) + 3(0) = 72$

At $(40, 0)$: $z = 2(40) + 3(0) = 80$

At $(24, 8)$: $z = 2(24) + 3(8) = 72$

The minimum value is 72 at any point on the line segment joining $(36, 0)$ and $(24, 8)$.

The maximum value is 80 at $(40, 0)$.



26. $z = 3x + 2y$

At $(0, 0)$: $z = 0$

At $(12, 0)$: $z = 36$

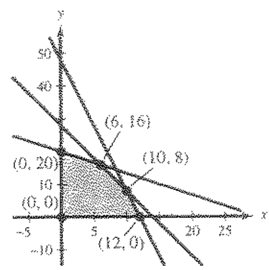
At $(10, 8)$: $z = 46$

At $(6, 16)$: $z = 50$

At $(0, 20)$: $z = 40$

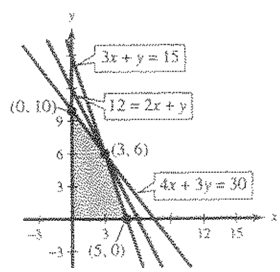
The maximum value is 50 at $(6, 16)$.

The minimum value is 0 at $(0, 0)$.



27. $z = 2x + y$

(a), (b)



(c) At $(0, 10)$: $z = 2(0) + (10) = 10$

At $(3, 6)$: $z = 2(3) + (6) = 12$

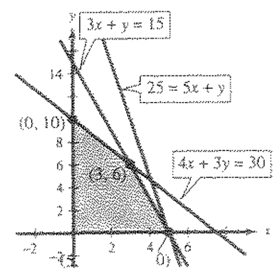
At $(5, 0)$: $z = 2(5) + (0) = 10$

At $(0, 0)$: $z = 2(0) + (0) = 0$

The maximum value is 12 at $(3, 6)$.

28. $z = 5x + y$

(a), (b)



(c) At $(0, 10)$: $z = 5(0) + (10) = 10$

At $(3, 6)$: $z = 5(3) + (6) = 21$

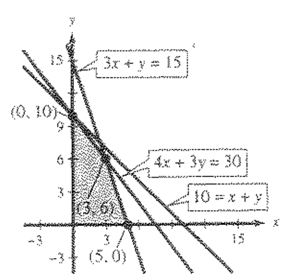
At $(5, 0)$: $z = 5(5) + (0) = 25$

At $(0, 0)$: $z = 5(0) + (0) = 0$

The maximum value is 25 at $(5, 0)$.

29. $z = x + y$

(a), (b)



(c) At $(0, 10)$: $z = (0) + (10) = 10$

At $(3, 6)$: $z = (3) + (6) = 9$

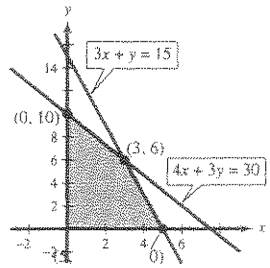
At $(5, 0)$: $z = (5) + (0) = 5$

At $(0, 0)$: $z = (0) + (0) = 0$

The maximum value is 10 at $(0, 10)$.

30. $z = 3x + y$

(a), (b)



(c) At (0, 10): $z = 3(0) + (10) = 10$

At (3, 6): $z = 3(3) + (6) = 15$

At (5, 0): $z = 3(5) + (0) = 15$

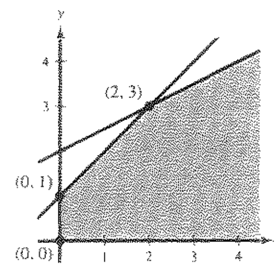
At (0, 0): $z = 3(0) + (0) = 0$

The maximum value is 15 at any point along the line segment connecting (3, 6) and (5, 0).

31. $-x + y \le 1 \implies y \le x + 1$

$-x + 2y \le 4 \implies y \le \frac{1}{2}x + 2$

Intersection: (2, 3)



The constraints do not form a closed set of points. Therefore, $z = x + y$ is unbounded.

32. Objective function: $z = 2.5x + y$

Constraints: $x \ge 0$, $y \ge 0$, $3x + 5y \le 15$, $5x + 2y \le 10$

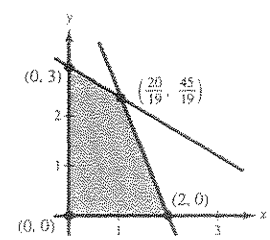
At (0, 0): $z = 2.5(0) + 0 = 0$

At (2, 0): $z = 2.5(2) + 0 = 5$

At $\left(\frac{20}{19}, \frac{45}{19}\right)$: $z = 2.5\left(\frac{20}{19}\right) + \frac{45}{19} = \frac{95}{19} = 5$

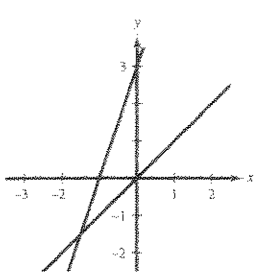
At (0, 3): $z = 2.5(0) + 3 = 3$

z is the maximum at any point on the line $5x + 2y = 10$ between the points (2, 0) and $\left(\frac{20}{19}, \frac{45}{19}\right)$.



33. $-x + y \le 0 \implies y \le x$

$-3x + y \ge 3 \implies y \ge 3x + 3$



The feasible set is empty.

34. Objective function: $z = -x + 2y$

Constraints: $x \ge 0$, $y \ge 0$, $x \le 10$, $x + y \le 7$

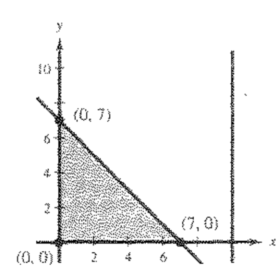
At (0, 0): $z = -0 + 2(0) = 0$

At (0, 7): $z = -0 + 2(7) = 14$

At (7, 0): $z = -7 + 2(0) = -7$

The constraint $x \le 10$ is extraneous.

The maximum value of 14 occurs at (0, 7).



35. Let x = number of audits.

Let y = number of tax returns.

Constraints: $100x + 12.5y \le 800$

$8x + 2y \le 96$

$x \ge 0$

$y \ge 0$

Objective function: $R = 2000x + 300y$

Vertices of feasible region: $(0, 0), (8, 0), (0, 48), (4, 32)$

At $(0, 0)$: $R = 0$

At $(8, 0)$: $R = 16{,}000$

At $(0, 48)$: $R = 14{,}400$

At $(4, 32)$: $R = 17{,}600$

4 audits, 32 tax returns yields maximum revenue of \$17,600.

36. x = amount of Model A

y = amount of Model B

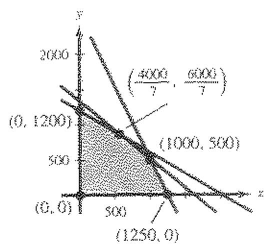
Constraints: $2.5x + 3y \le 4000$

$2x + y \le 2500$

$0.75x + 1.25y \le 1500$

$x \ge 0$

$y \ge 0$



Objective function: $P = 50x + 52y$

Vertices: $(0, 0), (0, 1200), \left(\frac{4000}{7}, \frac{6000}{7}\right), (1000, 500), (1250, 0)$

At $(0, 0)$: $P = (50)(0) + 52(0) = 0$

At $(0, 1200)$: $P = 50(0) + 52(1200) = 62{,}400$

At $\left(\frac{4000}{7}, \frac{6000}{7}\right)$: $P = 50\left(\frac{4000}{7}\right) + 52\left(\frac{6000}{7}\right) \approx 73{,}142.86$

At $(1000, 500)$: $P = 50(1000) + 52(500) = 76{,}000$

At $(1250, 0)$: $P = 50(1250) + 52(0) = 62{,}500$

The maximum profit (\$76,000) occurs when 1000 units of Model A and 500 units of Model B are produced.

37. x = number of bags of Brand X
y = number of bags of Brand Y

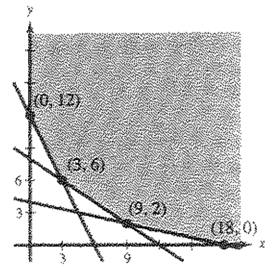
Constraints: $2x + y \geq 12$

$2x + 9y \geq 36$

$2x + 3y \geq 24$

$x \geq 0$

$y \geq 0$



Objective function: $C = 25x + 20y$

Vertices: $(0, 12), (3, 6), (9, 2), (18, 0)$

At $(0, 12)$: $C = 25(0) + 20(12) = 240$

At $(3, 6)$: $C = 25(3) + 20(6) = 195$

At $(9, 2)$: $C = 25(9) + 20(2) = 265$

At $(18, 0)$: $C = 25(18) + 20(0) = 450$

To minimize cost, use three bags of Brand X and six bags of Brand Y for a total cost of \$195.

38. x = number of bags of Brand X

y = number of bags of Brand Y

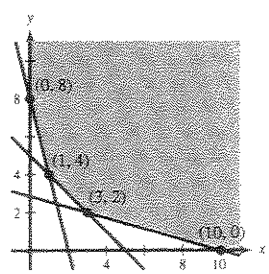
Constraints: $8x + 2y \geq 16$

$x + y \geq 5$

$2x + 7y \geq 20$

$x \geq 0$

$y \geq 0$



$C = 15x + 30y$

At $(0, 8)$: $C = 240$

At $(1, 4)$: $C = 135$

At $(3, 2)$: $C = 105$

At $(10, 0)$: $C = 150$

The minimum cost is \$105.

Use 3 bags of Brand X and 2 bags of Brand Y.

39. True, the maximum value is attained at all points in the segment joining these two vertices.

40. True, the maximum value is attained at all points on the segment joining these two vertices.

41. There are an infinite number of objective functions that would have a maximum at $(0, 4)$. One such objective function is $z = x + 5y$.

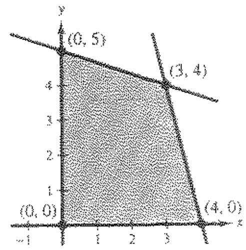
42. There are an infinite number of objective functions that would have a maximum at $(4, 3)$. One such objective function is $z = x + y$.

43. There are an infinite number of objective functions that would have a maximum at $(5, 0)$. One such objective function is $z = 4x + y$.

44. There are an infinite number of objective functions that would have a minimum at $(5, 0)$. One such objective function is $z = -10x + y$.

45. Constraints: $x \geq 0, y \geq 0, x + 3y \leq 15, 4x + y \leq 16$

Vertex	Value of $z = 3x + ty$
$(0, 0)$	$z = 0$
$(0, 5)$	$z = 5t$
$(3, 4)$	$z = 9 + 4t$
$(4, 0)$	$z = 12$



(a) For the maximum value to be at $(0, 5)$, $z = 5t$ must be greater than

$z = 9 + 4t$ and $z = 12$.

$5t > 9 + 4t$ and $5t > 12$

$t > 9$ $\qquad t > \frac{12}{5}$

Thus, $t > 9$.

(b) For the maximum value to be at $(3, 4)$, $z = 9 + 4t$ must be greater than $z = 5t$ and $z = 12$.

$9 + 4t > 5t$ and $9 + 4t > 12$

$9 > t$ $\qquad t > 3$

$t > \frac{3}{4}$

Thus, $\frac{3}{4} < t < 9$.

46. Constraints: $x \geq 0, y \geq 0, x + 2y \leq 4, x - y \leq 1$

$z = 3x + ty$

At $(0, 0)$: $z = 3(0) + t(0) = 0$

At $(1, 0)$: $z = 3(1) + t(0) = 3$

At $(2, 1)$: $z = 3(2) + t(1) = 6 + t$

At $(0, 2)$: $z = 3(0) + t(2) = 2t$

(a) For the maximum value to be at $(2, 1)$, $z = 6 + t$ must be greater than $z = 2t$ and $z = 3$.

$6 + t > 2t$ and $6 + t > 3$

$6 > t$ $\qquad t > -3$

Thus, $-3 < t < 6$.

(b) For maximum value to be at $(0, 2)$, $z = 2t$ must be greater than $z = 6 + t$ and $z = 3$.

$2t > 6 + t$ and $2t > 3$

$t > 6$ $\qquad t > \frac{3}{2}$

Thus, $t > 6$.

Chapter 1 Practice Test Solutions

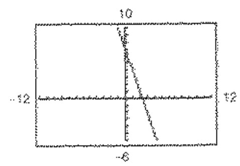
1. $m = \dfrac{3-2}{1-(-2)} = \dfrac{1}{3}$

2. Slope $= \dfrac{-2-(-5)}{3-4} = \dfrac{3}{-1} = -3$

$y + 2 = -3(x - 3)$

$y + 2 = -3x + 9$

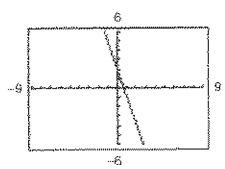
$y + 3x = 7$ or $y = -3x + 7$



3. $y - 5 = -3(x + 1)$

$y - 5 = -3x - 3$

$y + 3x = 2$ or $y = -3x + 2$



4. $3x + 5y = 7$

$5y = -3x + 7$

$y = -\frac{3}{5}x + \frac{7}{5}$

Slope of perpendicular line is $m = \frac{5}{3}$.

$y - 2 = \frac{5}{3}(x + 3)$

$y = \frac{5}{3}x + 7$

5. No, y is not a function of x. For example, $(0, 2)$ and $(0, -2)$ both satisfy the equation.

6. $f(0) = \dfrac{|0-2|}{(0-2)} = \dfrac{2}{-2} = -1$

$f(2)$ is not defined.

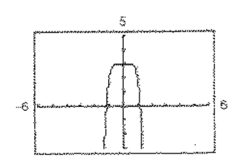
$f(4) = \dfrac{|4-2|}{(4-2)} = \dfrac{2}{2} = 1$

7. The domain of $f(x) = \dfrac{5}{x^2 - 16}$ is all $x \neq \pm 4$.

8. The domain of $g(t) = \sqrt{4 - t}$ consists of all t satisfying

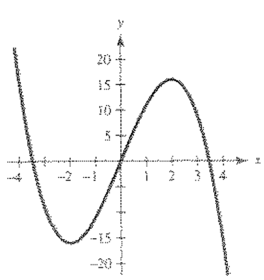
$4 - t \geq 0$ or $t \leq 4$.

9.



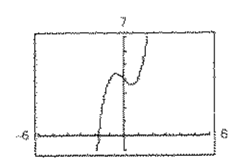
$f(x) = 3 - x^6$ is even.

10. $f(x) = 12x - x^3$



f is increasing on $(-2, 2)$.

11.



Relative minimum: $(0.577, 3.615)$

Relative maximum: $(-0.577, 4.385)$

12. $f(x) = x^3 - 3$ is a vertical shift of three units downward of $y = x^3$.

13. $f(x) = \sqrt{x - 6}$ is a horizontal shift six units to the right of $y = \sqrt{x}$.

14. $(g \circ f)(x) = g(f(x))$

$$= g(\sqrt{x}) = (\sqrt{x})^2 - 2 = x - 2$$

Domain: $x \geq 0$

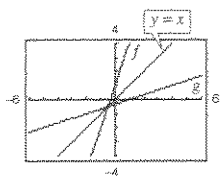
15. $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x^2}{16 - x^4}$

The domain is all $x \neq \pm 2$.

16. $(f \circ g)(x) = f\left(\dfrac{x-1}{3}\right)$

$$= 3\left(\frac{x-1}{3}\right) + 1 = (x - 1) + 1 = x$$

$$(g \circ f)(x) = g(3x + 1) = \frac{(3x + 1) - 1}{3} = \frac{3x}{3} = x$$



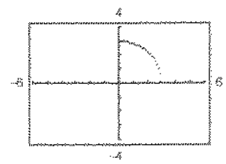
17. $y = \sqrt{9 - x^2}, \quad 0 \leq x \leq 3$

$x = \sqrt{9 - y^2}$

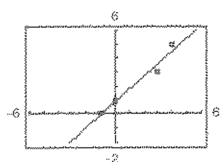
$x^2 = 9 - y^2$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$



18. $y = 0.882 + 0.912x$

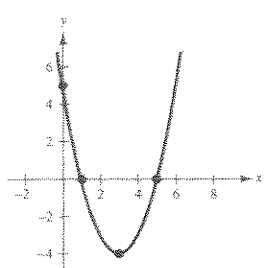


Chapter 2 Practice Test Solutions

1. x-intercepts: $(1, 0), (5, 0)$

y-intercept: $(0, 5)$

Vertex: $(3, -4)$



2. $a = 0.01, b = -90$

$$\frac{-b}{2a} = \frac{90}{2(.01)} = 4500 \text{ units}$$

3. Vertex: $(1, 7)$

Opening downward through $(2, 5)$

$y = a(x - 1)^2 + 7$ Standard form

$5 = a(2 - 1)^2 + 7$

$5 = a + 7$

$a = -2$

$y = -2(x - 1)^2 + 7 = -2(x^2 - 2x + 1) + 7 = -2x^2 + 4x + 5$

4. $y = \pm a(x - 2)(3x - 4)$ where a is any real number.

$y = \pm(3x^2 - 10x + 8)$

5. Leading coefficient: -3

Degree: 5

Moves down to the right and up to the left.

6. $0 = x^5 - 5x^3 + 4x$

$= x(x^4 - 5x^2 + 4)$

$= x(x^2 - 1)(x^2 - 4)$

$= x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, x = \pm 1, x = \pm 2$

7. $f(x) = x(x - 3)(x + 2)$

$= x(x^2 - x - 6)$

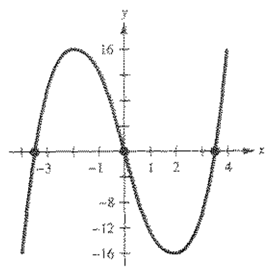
$= x^3 - x^2 - 6x$

8. Intercepts: $(0, 0)$, $(\pm 2\sqrt{3}, 0)$

Moves up to the right.

Moves down to the left.

x	-2	-1	0	1	2
y	16	11	0	-11	-16



9.

$$\begin{array}{r}
3x^3 + 9x^2 + 20x + 62 + \dfrac{176}{x-3} \\
x - 3\,\overline{)\,3x^4 + 0x^3 - 7x^2 + 2x - 10} \\
\underline{3x^4 - 9x^3} \\
9x^3 - 7x^2 \\
\underline{9x^3 - 27x^2} \\
20x^2 + 2x \\
\underline{20x^2 - 60x} \\
62x - 10 \\
\underline{62x - 186} \\
176
\end{array}$$

10.

$$\begin{array}{r}
x - 2 + \dfrac{5x - 13}{x^2 + 2x - 1} \\
x^2 + 2x - 1\,\overline{)\,x^3 + 0x^2 + 0x - 11} \\
\underline{x^3 + 2x^2 - x} \\
-2x^2 + x - 11 \\
\underline{-2x^2 - 4x + 2} \\
5x - 13
\end{array}$$

11.

$$\begin{array}{r|rrrrrr}
-5 & 3 & 13 & 0 & 0 & 12 & -1 \\
 & & -15 & 10 & -50 & 250 & -1310 \\
\hline
 & 3 & -2 & 10 & -50 & 262 & -1311
\end{array}$$

$$\frac{3x^5 + 13x^4 + 12x - 1}{x + 5} = 3x^4 - 2x^3 + 10x^2 - 50x + 262 - \frac{1311}{x + 5}$$

12.

$$\begin{array}{r|rrrr}
-6 & 7 & 40 & -12 & 15 \\
 & & -42 & 12 & 0 \\
\hline
 & 7 & -2 & 0 & 15
\end{array}$$

$f(-6) = 15$

13. $0 = x^3 - 19x - 30$

Possible rational roots:

$\pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -19 & -30 \\ & & -2 & 4 & 30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

-2 is a zero.

$0 = (x + 2)(x^2 - 2x - 15)$

$0 = (x + 2)(x + 3)(x - 5)$

Zeros: $x = -2, x = -3, x = 5$

14. $0 = x^4 + x^3 - 8x^2 - 9x - 9$

Possible rational roots: $\pm1, \pm3, \pm9$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -8 & -9 & -9 \\ & & 3 & 12 & 12 & 9 \\ \hline & 1 & 4 & 4 & 3 & 0 \end{array}$$

$x = 3$ is a zero.

$0 = (x - 3)(x^3 + 4x^2 + 4x + 3)$

Possible rational roots of $x^3 + 4x^2 + 4x + 3$: $\pm1, \pm3$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 4 & 3 \\ & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$x = -3$ is a zero.

$0 = (x - 3)(x + 3)(x^2 + x + 1)$

The zeros of $x^2 + x + 1$ are $x = \dfrac{-1 \pm \sqrt{3}i}{2}$.

Zeros: $x = 3, x = -3,$

$$x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ x = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

15. $0 = 6x^3 - 5x^2 + 4x - 15$

Possible rational roots: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

16. $0 = x^3 - \frac{20}{3}x^2 + 9x - \frac{10}{3}$

$0 = 3x^3 - 20x^2 + 27x - 10$

Possible rational roots:

$\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} 1 & 3 & -20 & 27 & -10 \\ & & 3 & -17 & 10 \\ \hline & 3 & -17 & 10 & 0 \end{array}$$

$x = 1$ is a zero.

$0 = (x - 1)(3x^2 - 17x + 10)$

$0 = (x - 1)(3x - 2)(x - 5)$

Zeros: $x = 1, x = \frac{2}{3}, x = 5$

17. $f(x) = x^4 + x^3 + 3x^2 + 5x - 10$

Possible rational roots: $\pm1, \pm2, \pm5, \pm10$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 3 & 5 & -10 \\ & & 1 & 2 & 5 & 10 \\ \hline & 1 & 2 & 5 & 10 & 0 \end{array}$$

$x = 1$ is a zero.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 5 & 10 \\ & & -2 & 0 & -10 \\ \hline & 1 & 0 & 5 & 0 \end{array}$$

$x = -2$ is a zero.

$f(x) = (x - 1)(x + 2)(x^2 + 5)$

$\quad = (x - 1)(x + 2)(x + \sqrt{5}i)(x - \sqrt{5}i)$

18. $\dfrac{2}{1 + i} = \dfrac{2}{1 + i} \cdot \dfrac{1 - i}{1 - i}$

$= \dfrac{2 - 2i}{1 + 1}$

$= 1 - i$

19. $\dfrac{3 + i}{2} - \dfrac{i + 1}{4} = \dfrac{6 + 2i - i - 1}{4} = \dfrac{5}{4} + \dfrac{1}{4}i$

20. $f(x) = (x-2)[x-(3+i)][x-(3-i)][x-(3-2i)][x-(3+2i)]$

$= (x-2)[(x-3)^2+1][(x-3)^2+4]$

$= (x-2)(x^2-6x+10)(x^2-6x+13)$

$= x^5 - 14x^4 + 83x^3 - 256x^2 + 406x - 260$

21.
$$\begin{array}{r|rrrr} 3i & 1 & 4 & 9 & 36 \\ & & 3i & 12i-9 & -36 \\ \hline & 1 & 4+3i & 12i & 0 \end{array}$$

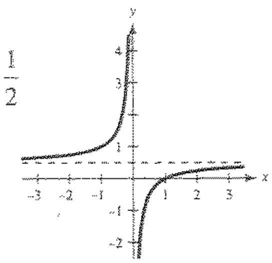
22. $z = \dfrac{kx^2}{\sqrt{y}}$

23. $f(x) = \dfrac{x-1}{2x}$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = \dfrac{1}{2}$

x-intercept: $(1, 0)$

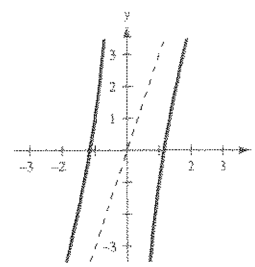


24. $f(x) = \dfrac{3x^2-4}{x}$

Vertical asymptote: $x = 0$

Slant asymptote: $y = 3x$

x-intercepts: $\left(\pm\dfrac{2}{\sqrt{3}}, 0\right)$



25. $y = 8$ is a horizontal asymptote since the degree of the numerator equals the degree of the denominator. There are no vertical asymptotes.

26. $x = 1$ is a vertical asymptote.

$$\frac{4x^2-2x+7}{x-1} = 4x+2+\frac{9}{x-1}$$

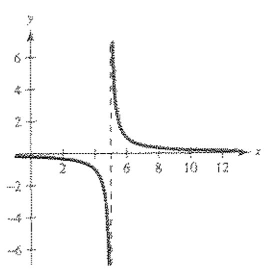
so $y = 4x + 2$ is a slant asymptote.

27. $f(x) = \dfrac{x-5}{(x-5)^2} = \dfrac{1}{x-5}$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = 0$

y-intercept: $\left(0, -\dfrac{1}{5}\right)$



Chapter 3 Practice Test Solutions

1. $x^{3/5} = 8$

$x = 8^{5/3}$

$= \left(\sqrt[3]{8}\right)^5 = 2^5 = 32$

2. $3^{x-1} = \frac{1}{81}$

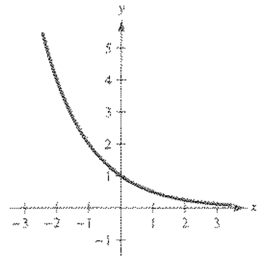
$3^{x-1} = 3^{-4}$

$x - 1 = -4$

$x = -3$

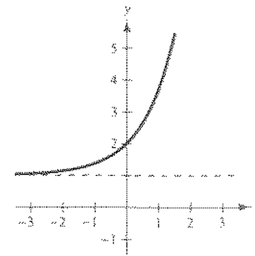
3. $f(x) = 2^{-x} = \left(\frac{1}{2}\right)^x$

x	-2	-1	0	1	2
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



4. $g(x) = e^x + 1$

x	-2	-1	0	1	2
$g(x)$	1.14	1.37	2	3.72	8.39



5. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

(a) $A = 5000\left(1 + \frac{0.09}{12}\right)^{12(3)} \approx \$6543.23$

(b) $A = 5000\left(1 + \frac{0.09}{4}\right)^{4(3)} \approx \$6530.25$

(c) $A = 5000e^{(0.09)(3)} \approx \$6549.82$

6. $7^{-2} = \frac{1}{49}$

$\log_7 \frac{1}{49} = -2$

7.
$$\begin{aligned} x - 4 &= \log_2 \tfrac{1}{64} \\ 2^{x-4} &= \tfrac{1}{64} \\ 2^{x-4} &= 2^{-6} \\ x - 4 &= -6 \\ x &= -2 \end{aligned}$$

8.
$$\begin{aligned} \log_b \sqrt[4]{\tfrac{8}{25}} &= \tfrac{1}{4}\log_b \tfrac{8}{25} \\ &= \tfrac{1}{4}[\log_b 8 - \log_b 25] \\ &= \tfrac{1}{4}[\log_b 2^3 - \log_b 5^2] \\ &= \tfrac{1}{4}[3\log_b 2 - 2\log_b 5] \\ &= \tfrac{1}{4}[3(0.3562) - 2(0.8271)] \\ &= -0.1464 \end{aligned}$$

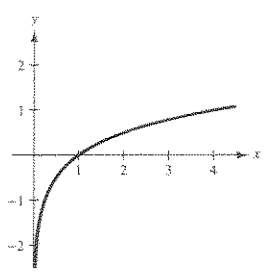
9. $5\ln x - \frac{1}{2}\ln y + 6\ln z = \ln x^5 - \ln\sqrt{y} + \ln z^6 = \ln\left(\frac{x^5z^6}{\sqrt{y}}\right)$

10. $\log_9 28 = \frac{\log 28}{\log 9} \approx 1.5166$

11. $\log_{10} N = 0.6646$

$N = 10^{0.6646} \approx 4.62$

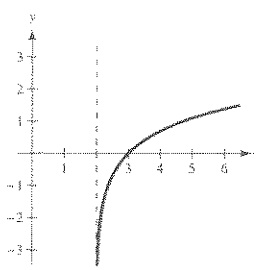
12.



13. Domain:

$$\begin{aligned} x^2 - 9 &> 0 \\ (x + 3)(x - 3) &> 0 \\ x < -3 \text{ or } x &> 3 \end{aligned}$$

14.



15. $\dfrac{\ln x}{\ln y} \neq \ln(x - y)$ since $\dfrac{\ln x}{\ln y} = \log_y x$.

16. $5^x = 41$

$$x = \log_5 41 = \frac{\ln 41}{\ln 5} \approx 2.3074$$

17. $x - x^2 = \log_5 \frac{1}{25}$

$$5^{x-x^2} = \tfrac{1}{25}$$
$$5^{x-x^2} = 5^{-2}$$
$$x - x^2 = -2$$
$$0 = x^2 - x - 2$$
$$0 = (x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

18. $\log_2 x + \log_2(x - 3) = 2$

$$\log_2[x(x - 3)] = 2$$
$$x(x - 3) = 2^2$$
$$x^2 - 3x = 4$$
$$x^2 - 3x - 4 = 0$$
$$(x + 1)(x - 4) = 0$$
$$x = 4$$
$$x = -1 \quad \text{(extraneous solution)}$$

19. $\dfrac{e^x + e^{-x}}{3} = 4$

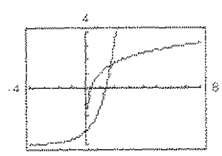
$$e^x(e^x + e^{-x}) = 12e^x$$
$$e^{2x} + 1 = 12e^x$$
$$e^{2x} - 12e^x + 1 = 0$$
$$e^x = \frac{12 \pm \sqrt{144 - 4}}{2}$$
$$e^x \approx 11.9161 \quad \text{or} \quad e^x \approx 0.08392$$
$$x \approx \ln 11.9161 \qquad x \approx \ln 0.08392$$
$$x \approx 2.4779 \qquad x \approx -2.4779$$

20. $A = Pe^{rt}$

$$12{,}000 = 6000e^{0.13t}$$
$$2 = e^{0.13t}$$
$$\ln 2 = 0.13t$$
$$\frac{\ln 2}{0.13} = t$$
$$t \approx 5.3319 \text{ yr or } 5 \text{ yr } 4 \text{ mo}$$

21. There are two points of intersection:

$(0.0169, -2.983)$,
$(1.731, 1.647)$



22. $y = 1.0597x^{1.9792}$

Chapter 4 Practice Test Solutions

1. $350° = 350°\left(\dfrac{\pi}{180°}\right) = \dfrac{35\pi}{18}$

2. $\dfrac{5\pi}{9} = \dfrac{5\pi}{9} \cdot \dfrac{180°}{\pi} = 100°$

3. $135°\ 14'\ 12'' = \left(135 + \frac{14}{60} + \frac{12}{3600}\right)°$

$$\approx 135.2367°$$

4. $-22.569° = -(22° + 0.569(60)')$

$$= -22°\ 34.14'$$
$$= -(22°\ 34' + 0.14(60)'')$$
$$\approx -22°\ 34'\ 8''$$

5. $\cos\theta = \dfrac{2}{3}$

$x = 2,\ r = 3,\ y = \pm\sqrt{9-4} = \pm\sqrt{5}$

$\tan\theta = \dfrac{y}{x} = \pm\dfrac{\sqrt{5}}{2}$

6. $\sin\theta = 0.9063$

$\theta = \arcsin 0.9063$

$\theta \approx 65^\circ \text{ or } \dfrac{13\pi}{36}$

7. $\tan 20^\circ = \dfrac{35}{x}$

$x = \dfrac{35}{\tan 20^\circ} \approx 96.1617$

8. $\theta = \dfrac{6\pi}{5}$, θ is in Quadrant III.

Reference angle: $\dfrac{6\pi}{5} - \pi = \dfrac{\pi}{5}$ or 36°

9. $\csc 3.92 = \dfrac{1}{\sin 3.92} \approx -1.4242$

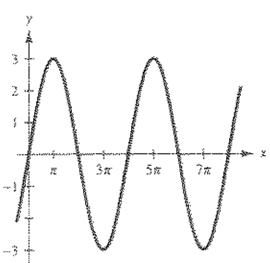
10. $\tan\theta = 6 = \dfrac{6}{1}$, θ lies in Quadrant III.

$y = -6,\ x = -1,\ r = \sqrt{36+1} = \sqrt{37}$, so

$\sec\theta = \dfrac{\sqrt{37}}{-1} \approx -6.0828.$

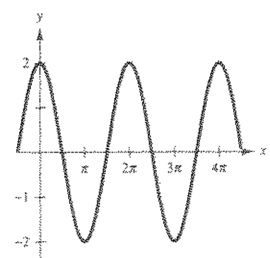
11. Period: 4π

Amplitude: 3

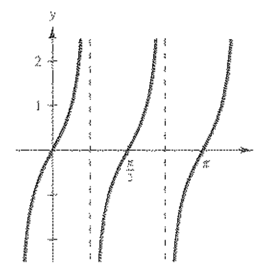


12. Period: 2π

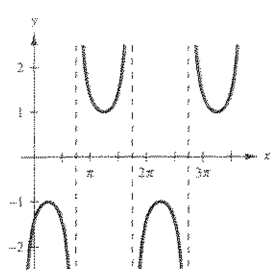
Amplitude: 2



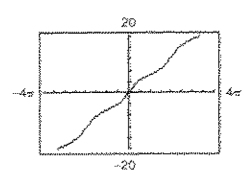
13. Period: $\dfrac{\pi}{2}$



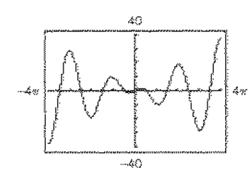
14. Period: 2π



15.



16.



17. $\theta = \arcsin 1$

$\sin\theta = 1$

$\theta = \dfrac{\pi}{2}$

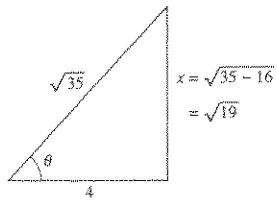
18. $\theta = \arctan(-3)$

$\tan\theta = -3$

$\theta \approx -1.249 \text{ or } -71.565^\circ$

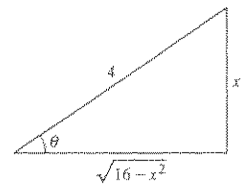
19. $\sin\left(\arccos \dfrac{4}{\sqrt{35}}\right)$

$\sin \theta = \dfrac{\sqrt{19}}{\sqrt{35}} \approx 0.7368$



20. $\cos\left(\arcsin \dfrac{x}{4}\right)$

$\cos \theta = \dfrac{\sqrt{16 - x^2}}{4}$



21. Given $A = 40°$, $c = 12$

$B = 90° - 40° = 50°$

$\sin 40° = \dfrac{a}{12}$

$a = 12 \sin 40° \approx 7.713$

$\cos 40° = \dfrac{b}{12}$

$b = 12 \cos 40° \approx 9.193$

22. Given $B = 6.84°$, $a = 21.3$

$A = 90° - 6.84° = 83.16°$

$\sin 83.16° = \dfrac{21.3}{c}$

$c = \dfrac{21.3}{\sin 83.16°} \approx 21.453$

$\tan 83.16° = \dfrac{21.3}{b}$

$b = \dfrac{21.3}{\tan 83.16°} \approx 2.555$

23. Given $a = 5$, $b = 9$

$c = \sqrt{25 + 81} = \sqrt{106}$

≈ 10.296

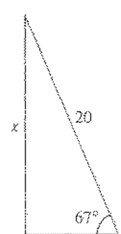
$\tan A = \dfrac{5}{9}$

$A = \arctan \dfrac{5}{9} \approx 29.055°$

$B = 90° - 29.055° = 60.945°$

24. $\sin 67° = \dfrac{x}{20}$

$x = 20 \sin 67° \approx 18.41$ feet

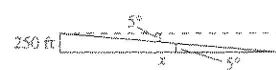


25. $\tan 5° = \dfrac{250}{x}$

$x = \dfrac{250}{\tan 5°}$

≈ 2857.513 feet

≈ 0.541 mi



Chapter 5 Practice Test Solutions

1. $\tan x = \dfrac{4}{11}$, $\sec x < 0 \implies x$ is in Quadrant III.

$y = -4$, $x = -11$, $r = \sqrt{16 + 121} = \sqrt{137}$

$\sin x = -\dfrac{4}{\sqrt{137}} = -\dfrac{4\sqrt{137}}{137}$ $\qquad \csc x = -\dfrac{\sqrt{137}}{4}$

$\cos x = -\dfrac{11}{\sqrt{137}} = -\dfrac{11\sqrt{137}}{137}$ $\qquad \sec x = -\dfrac{\sqrt{137}}{11}$

$\tan x = \dfrac{4}{11}$ $\qquad \cot x = \dfrac{11}{4}$

2.
$$\frac{\sec^2 x + \csc^2 x}{\csc^2 x(1 + \tan^2 x)} = \frac{\sec^2 x + \csc^2 x}{\csc^2 x + (\csc^2 x)\tan^2 x}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \dfrac{1}{\sin^2 x} \cdot \dfrac{\sin^2 x}{\cos^2 x}}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \dfrac{1}{\cos^2 x}}$$
$$= \frac{\sec^2 x + \csc^2 x}{\csc^2 x + \sec^2 x} = 1$$

3. $\ln|\tan\theta| - \ln|\cot\theta| = \ln\dfrac{|\tan\theta|}{|\cot\theta|}$

$= \ln\left|\dfrac{\sin\theta/\cos\theta}{\cos\theta/\sin\theta}\right| = \ln\left|\dfrac{\sin^2\theta}{\cos^2\theta}\right|$

$= \ln|\tan^2\theta| = 2\ln|\tan\theta|$

4. $\cos\left(\dfrac{\pi}{2} - x\right) = \dfrac{1}{\csc x}$ is true since

$\cos\left(\dfrac{\pi}{2} - x\right) = \sin x = \dfrac{1}{\csc x}.$

5. $\sin^4 x + (\sin^2 x)\cos^2 x = \sin^2 x(\sin^2 x + \cos^2 x)$

$= \sin^2 x(1) = \sin^2 x$

6. $(\csc x + 1)(\csc x - 1) = \csc^2 x - 1 = \cot^2 x$

7. $\dfrac{\cos^2 x}{1 - \sin x}\cdot\dfrac{1 + \sin x}{1 + \sin x} = \dfrac{\cos^2 x(1 + \sin x)}{1 - \sin^2 x} = \dfrac{\cos^2 x(1 + \sin x)}{\cos^2 x} = 1 + \sin x$

8. $\dfrac{1 + \cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{(1 + \cos\theta)^2 + \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \dfrac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

$= \dfrac{2 + 2\cos\theta}{\sin\theta(1 + \cos\theta)} = \dfrac{2}{\sin\theta} = 2\csc\theta$

9. $\tan^4 x + 2\tan^2 x + 1 = (\tan^2 x + 1)^2 = (\sec^2 x)^2 = \sec^4 x$

10. (a) $\sin 105° = \sin(60° + 45°)$

$= \sin 60°\cos 45° + \cos 60°\sin 45°$

$= \dfrac{\sqrt{3}}{2}\cdot\dfrac{\sqrt{2}}{2} + \dfrac{1}{2}\cdot\dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}}{4}\left(\sqrt{3} + 1\right)$

(b) $\tan 15° = \tan(60° - 45°) = \dfrac{\tan 60° - \tan 45°}{1 + \tan 60°\tan 45°}$

$= \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}\cdot\dfrac{1 - \sqrt{3}}{1 - \sqrt{3}} = \dfrac{2\sqrt{3} - 1 - 3}{1 - 3}$

$= \dfrac{2\sqrt{3} - 4}{-2} = 2 - \sqrt{3}$

11. $(\sin 42°)\cos 38° - (\cos 42°)\sin 38° = \sin(42° - 38°) = \sin 4°$

12. $\tan\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{\tan\theta + \tan(\pi/4)}{1 - (\tan\theta)\tan(\pi/4)} = \dfrac{\tan\theta + 1}{1 - \tan\theta(1)} = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

13. $\sin(\arcsin x - \arccos x) = \sin(\arcsin x)\cos(\arccos x) - \cos(\arcsin x)\sin(\arccos x)$

$= (x)(x) - \left(\sqrt{1 - x^2}\right)\left(\sqrt{1 - x^2}\right) = x^2 - (1 - x^2) = 2x^2 - 1$

14. (a) $\cos(120°) = \cos[2(60°)] = 2\cos^2 60° - 1 = 2\left(\dfrac{1}{2}\right)^2 - 1 = -\dfrac{1}{2}$

(b) $\tan(300°) = \tan[2(150°)] = \dfrac{2\tan 150°}{1 - \tan^2 150°} = \dfrac{-2\sqrt{3}/3}{1 - (1/3)} = -\sqrt{3}$

15. (a) $\sin 22.5° = \sin \dfrac{45°}{2} = \sqrt{\dfrac{1 - \cos 45°}{2}} = \sqrt{\dfrac{1 - \sqrt{2}/2}{2}} = \dfrac{\sqrt{2 - \sqrt{2}}}{2}$

(b) $\tan \dfrac{\pi}{12} = \tan \dfrac{\pi/6}{2} = \dfrac{\sin(\pi/6)}{1 + \cos(\pi/6)} = \dfrac{1/2}{1 + \sqrt{3}/2} = \dfrac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$

16. $\sin \theta = \dfrac{4}{5}$, θ lies in Quadrant II $\implies \cos \theta = -\dfrac{3}{5}$.

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 - (3/5)}{2}}$$
$$= \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

17. $(\sin^2 x) \cos^2 x = \dfrac{1 - \cos 2x}{2} \cdot \dfrac{1 + \cos 2x}{2}$

$$= \frac{1}{4}[1 - \cos^2 2x]$$
$$= \frac{1}{4}\left[1 - \frac{1 + \cos 4x}{2}\right]$$
$$= \frac{1}{8}[2 - (1 + \cos 4x)]$$
$$= \frac{1}{8}[1 - \cos 4x]$$

18. $6(\sin 5\theta) \cos 2\theta = 6\{\tfrac{1}{2}[\sin(5\theta + 2\theta) + \sin(5\theta - 2\theta)]\} = 3[\sin 7\theta + \sin 3\theta]$

19. $\sin(x + \pi) + \sin(x - \pi) = 2\left(\sin \dfrac{[(x + \pi) + (x - \pi)]}{2}\right) \cos \dfrac{[(x + \pi) - (x - \pi)]}{2} = 2 \sin x \cos \pi = -2 \sin x$

20. $\dfrac{\sin 9x + \sin 5x}{\cos 9x - \cos 5x} = \dfrac{2 \sin 7x \cos 2x}{-2 \sin 7x \sin 2x} = -\dfrac{\cos 2x}{\sin 2x} = -\cot 2x$

21. $\tfrac{1}{2}[\sin(u + v) - \sin(u - v)] = \tfrac{1}{2}\{(\sin u) \cos v + (\cos u) \sin v - [(\sin u) \cos v - (\cos u) \sin v]\}$

$$= \tfrac{1}{2}[2(\cos u) \sin v] = (\cos u) \sin v$$

22. $4 \sin^2 x = 1$

$$\sin^2 x = \frac{1}{4}$$
$$\sin x = \pm\frac{1}{2}$$

$\sin x = \dfrac{1}{2}$ or $\sin x = -\dfrac{1}{2}$

$x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ $\qquad$ $x = \dfrac{7\pi}{6}$ or $\dfrac{11\pi}{6}$

23. $\tan^2 \theta + \left(\sqrt{3} - 1\right) \tan \theta - \sqrt{3} = 0$

$$(\tan \theta - 1)\left(\tan \theta + \sqrt{3}\right) = 0$$

$\tan \theta = 1$ or $\tan \theta = -\sqrt{3}$

$\theta = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$ $\qquad$ $\theta = \dfrac{2\pi}{3}$ or $\dfrac{5\pi}{3}$

24. $\sin 2x = \cos x$

$$2(\sin x) \cos x - \cos x = 0$$
$$\cos x(2 \sin x - 1) = 0$$

$\cos x = 0$ or $\sin x = \dfrac{1}{2}$

$x = \dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$ $\qquad$ $x = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

25. $\tan^2 x - 6\tan x + 4 = 0$

$$\tan x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$

$$\tan x = \frac{6 \pm \sqrt{20}}{2} = 3 \pm \sqrt{5}$$

$\tan x = 3 + \sqrt{5}$ or $\tan x = 3 - \sqrt{5}$

$x \approx 1.3821$ or 4.5237 $\quad x = 0.6524$ or 3.7940

Chapter 6 Practice Test Solutions

1. $C = 180° - (40° + 12°) = 128°$

$a = \sin 40°\left(\dfrac{100}{\sin 12°}\right) \approx 309.164$

$c = \sin 128°\left(\dfrac{100}{\sin 12°}\right) \approx 379.012$

2. $\sin A = 5\left(\dfrac{\sin 150°}{20}\right) = 0.125$

$A \approx 7.181°$

$B \approx 180° - (150° + 7.181°) = 22.819°$

$b = \sin 22.819°\left(\dfrac{20}{\sin 150°}\right) \approx 15.513$

3. Area $= \frac{1}{2}ab \sin C$

$= \frac{1}{2}(3)(6)\sin 130°$

≈ 6.894 square units

4. $h = b \sin A = 35 \sin 22.5° \approx 13.394$

$a = 10$

Since $a < h$ and A is acute, the triangle has no solution.

5. $\cos A = \dfrac{(53)^2 + (38)^2 - (49)^2}{2(53)(38)} \approx 0.4598$

$A \approx 62.627°$

$\cos B = \dfrac{(49)^2 + (38)^2 - (53)^2}{2(49)(38)} \approx 0.2782$

$B \approx 73.847°$

$C \approx 180° - (62.627° + 73.847°) = 43.526°$

6. $c^2 = (100)^2 + (300)^2 - 2(100)(300)\cos 29°$

$\approx 47{,}522.8176$

$c \approx 218$

$\cos A = \dfrac{(300)^2 + (218)^2 - (100)^2}{2(300)(218)} \approx 0.97495$

$A \approx 12.85°$

$B \approx 180° - (12.85° + 29°) = 138.15°$

7. $s = \dfrac{a + b + c}{2} = \dfrac{4.1 + 6.8 + 5.5}{2} = 8.2$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

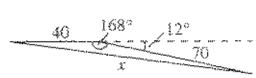
$= \sqrt{8.2(8.2 - 4.1)(8.2 - 6.8)(8.2 - 5.5)}$

$= 11.273$ square units

8. $x^2 = (40)^2 + (70)^2 - 2(40)(70)\cos 168°$

$\approx 11{,}977.6266$

$x \approx 190.442$ miles



9. $\mathbf{w} = 4(3\mathbf{i} + \mathbf{j}) - 7(-\mathbf{i} + 2\mathbf{j}) = 19\mathbf{i} - 10\mathbf{j}$

10. $\dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{5\mathbf{i} - 3\mathbf{j}}{\sqrt{25 + 9}} = \dfrac{5}{\sqrt{34}}\mathbf{i} - \dfrac{3}{\sqrt{34}}\mathbf{j}$

$= \dfrac{5\sqrt{34}}{34}\mathbf{i} - \dfrac{3\sqrt{34}}{34}\mathbf{j}$

11. $\mathbf{u} = 6\mathbf{i} + 5\mathbf{j},\ \mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$

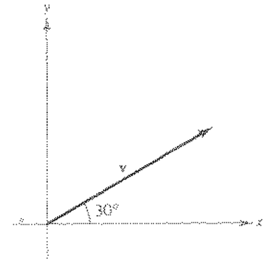
$\mathbf{u} \cdot \mathbf{v} = 6(2) + 5(-3) = -3$

$\|\mathbf{u}\| = \sqrt{61},\ \|\mathbf{v}\| = \sqrt{13}$

$\cos\theta = \dfrac{-3}{\sqrt{61}\sqrt{13}}$

$\theta \approx 96.116^\circ$

12. $4(\mathbf{i}\cos 30^\circ + \mathbf{j}\sin 30^\circ) = 4\left(\dfrac{\sqrt{3}}{2}\mathbf{i} + \dfrac{1}{2}\mathbf{j}\right) = \langle 4\sqrt{3}, 2\rangle$



13. $\operatorname{proj}_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\right)\mathbf{v} = \dfrac{-10}{20}\langle -2, 4\rangle = \langle 1, -2\rangle$

14. $r = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$

$\tan\theta = \dfrac{-5}{5} = -1$

Since z is in Quadrant IV,

$\theta = 315^\circ$

$z = 5\sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$.

15. $\cos 225^\circ = -\dfrac{\sqrt{2}}{2}\ \sin 225^\circ = -\dfrac{\sqrt{2}}{2}$

$z = 6\left(-\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}\right)$

$= -3\sqrt{2} - 3\sqrt{2}i$

16. $[7(\cos 23^\circ + i\sin 23^\circ)][4(\cos 7^\circ + i\sin 7^\circ)] = 7(4)[\cos(23^\circ + 7^\circ) + i\sin(23^\circ + 7^\circ)]$

$= 28(\cos 30^\circ + i\sin 30^\circ) = 14\sqrt{3} + 14i$

17. $\dfrac{9\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)}{3(\cos\pi + i\sin\pi)} = \dfrac{9}{3}\left[\cos\left(\dfrac{5\pi}{4} - \pi\right) + i\sin\left(\dfrac{5\pi}{4} - \pi\right)\right] = 3\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right) = \dfrac{3\sqrt{2}}{2} + \dfrac{3\sqrt{2}}{2}i$

18. $(2 + 2i)^8 = [2\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)]^8 = (2\sqrt{2})^8[\cos(8)(45^\circ) + i\sin(8)(45^\circ)]$

$= 4096[\cos 360^\circ + i\sin 360^\circ] = 4096$

19. $z = 8\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right),\ n = 3$

The cube roots of z are:

For $k = 0$, $\sqrt[3]{8}\left[\cos\dfrac{\pi/3}{3} + i\sin\dfrac{\pi/3}{3}\right] = 2\left(\cos\dfrac{\pi}{9} + i\sin\dfrac{\pi}{9}\right)$.

For $k = 1$, $\sqrt[3]{8}\left[\cos\dfrac{\pi/3 + 2\pi}{3} + i\sin\dfrac{\pi/3 + 2\pi}{3}\right] = 2\left(\cos\dfrac{7\pi}{9} + i\sin\dfrac{7\pi}{9}\right)$.

For $k = 2$, $\sqrt[3]{8}\left[\cos\dfrac{\pi/3 + 4\pi}{3} + i\sin\dfrac{\pi/3 + 4\pi}{3}\right] = 2\left(\cos\dfrac{13\pi}{9} + i\sin\dfrac{13\pi}{9}\right)$.

20. $x^4 = -i = 1\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$

For $k = 0$, $\cos\frac{3\pi/2}{4} + i\sin\frac{3\pi/2}{4} = \cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}$.

For $k = 1$, $\cos\frac{3\pi/2 + 2\pi}{4} + i\sin\frac{3\pi/2 + 2\pi}{4} = \cos\frac{7\pi}{8} + i\sin\frac{7\pi}{8}$.

For $k = 2$, $\cos\frac{3\pi/2 + 4\pi}{4} + i\sin\frac{3\pi/2 + 4\pi}{4} = \cos\frac{11\pi}{8} + i\sin\frac{11\pi}{8}$.

For $k = 3$, $\cos\frac{3\pi/2 + 6\pi}{4} + i\sin\frac{3\pi/2 + 6\pi}{4} = \cos\frac{15\pi}{8} + i\sin\frac{15\pi}{8}$.

Chapter 7 Practice Test Solutions

1. $\begin{cases} x + y = 1 \\ 3x - y = 15 \implies y = 3x - 15 \end{cases}$

$$\begin{aligned} x + (3x - 15) &= 1 \\ 4x &= 16 \\ x &= 4 \\ y &= -3 \end{aligned}$$

2. $\begin{cases} x - 3y = -3 \implies x = 3y - 3 \\ x^2 + 6y = 5 \end{cases}$

$$\begin{aligned} (3y - 3)^2 + 6y &= 5 \\ 9y^2 - 18y + 9 + 6y &= 5 \\ 9y^2 - 12y + 4 &= 0 \\ (3y - 2)^2 &= 0 \\ y &= \tfrac{2}{3} \\ x &= -1 \end{aligned}$$

3. $\begin{cases} x + y + z = 6 \implies z = 6 - x - y \\ 2x - y + 3z = 0 \qquad 2x - y + 3(6 - x - y) = 0 \implies -x - 4y = -18 \\ 5x + 2y - z = -3 \qquad 5x + 2y - (6 - x - y) = -3 \implies 6x + 3y = 3 \end{cases}$

$$\begin{aligned} x &= 18 - 4y \\ 6(18 - 4y) + 3y &= 3 \\ -21y &= -105 \\ y &= 5 \\ x &= 18 - 4y = -2 \\ z &= 6 - x - y = 3 \end{aligned}$$

4. $\begin{cases} x + y = 110 \implies y = 110 - x \\ xy = 2800 \end{cases}$

$$\begin{aligned} x(110 - x) &= 2800 \\ 0 &= x^2 - 110x + 2800 \\ 0 &= (x - 40)(x - 70) \\ x &= 40 \quad\text{or}\quad x = 70 \\ y &= 70 \qquad\quad\; y = 40 \end{aligned}$$

5. $\begin{cases} 2x + 2y = 170 \implies y = \frac{170 - 2x}{2} = 85 - x \\ xy = 1500 \end{cases}$

$$\begin{aligned} x(85 - x) &= 1500 \\ 0 &= x^2 - 85x + 1500 \\ 0 &= (x - 25)(x - 60) \\ x &= 25 \quad\text{or}\quad x = 60 \\ y &= 60 \qquad\quad\; y = 25 \end{aligned}$$

Dimensions: $60' \times 25'$

6. $\begin{cases} 2x + 15y = 4 \\ x - 3y = 23 \end{cases} \begin{array}{l} \Longrightarrow \\ \Longrightarrow \end{array} \begin{array}{r} 2x + 15y = 4 \\ \underline{5x - 15y = 115} \\ 7x \quad\quad = 119 \end{array}$

$$x = 17$$

$$y = \frac{x - 23}{3} = -2$$

7. $\begin{cases} x + y = 2 \\ 38x - 19y = 7 \end{cases} \begin{array}{l} \Longrightarrow \\ \Longrightarrow \end{array} \begin{array}{r} 19x + 19y = 38 \\ \underline{38x - 19y = 7} \\ 57x \quad\quad = 45 \end{array}$

$$x = \frac{45}{57} = \frac{15}{19}$$

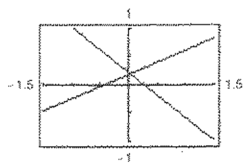
$$y = 2 - x$$

$$= \frac{38}{19} - \frac{15}{19}$$

$$= \frac{23}{19}$$

8. $y_1 = 2(0.112 - 0.4x)$

$$y_2 = \frac{(0.131 + 0.3x)}{0.7}$$



$\begin{cases} 0.4x + 0.5y = 0.112 \\ 0.3x - 0.7y = -0.131 \end{cases} \begin{array}{l} \Longrightarrow \\ \Longrightarrow \end{array} \begin{array}{r} 0.28x + 0.35y = 0.0784 \\ \underline{0.15x - 0.35y = -0.0655} \\ 0.43x \quad\quad = 0.0129 \end{array}$

$$x = \frac{0.0129}{0.43} = 0.03$$

$$y = (2)(0.112 - 0.4x) = 0.20$$

9. Let x = amount in 11% fund and y = amount in 13% fund.

$$\begin{cases} x + y = 17{,}000 \Longrightarrow y = 17{,}000 - x \\ 0.11x + 0.13y = 2080 \end{cases}$$

$$0.11x + 0.13(17{,}000 - x) = 2080$$

$$-0.02x = -130$$

$$x = \$6500$$

$$y = \$10{,}500$$

10. Using a graphing utility, you obtain $y = 0.7857x - 0.1429$. Analytically, $(4, 3)$, $(1, 1)$, $(-1, -2)$, $(-2, -1)$.

$$n = 4,\ \sum_{i=1}^{4} x_i = 2,\ \sum_{i=1}^{4} y_i = 1,\ \sum_{i=1}^{4} x_i^2 = 22,\ \sum_{i=1}^{4} x_i y_i = 17$$

$\begin{array}{l} 4b + 2a = 1 \\ 2b + 22a = 17 \end{array} \begin{array}{l} \Longrightarrow \\ \Longrightarrow \end{array} \begin{array}{r} 4b + 2a = 1 \\ \underline{-4b - 44a = -34} \\ -42a = -33 \end{array}$

$$a = \tfrac{33}{42} = \tfrac{11}{14}$$

$$b = \tfrac{1}{4}\left(1 - 2\left(\tfrac{33}{42}\right)\right) = -\tfrac{1}{7}$$

$$y = ax + b = \tfrac{11}{14}x - \tfrac{1}{7}$$

11. $\begin{array}{rl} x + y \quad\quad = -2 & \text{Equation 1} \\ 2x - y + z = 11 & \text{Equation 2} \\ 4y - 3z = -20 & \text{Equation 3} \end{array}$

$$\begin{cases} x + y \quad = -2 \\ -3y + z = 15 & -2\text{Eq.}1 + \text{Eq.}2 \\ 4y - 3z = -20 \end{cases}$$

$$\begin{cases} x + y \quad = -2 \\ -3y + z = 15 \\ -5y \quad = 25 & 3\text{Eq.}2 + \text{Eq.}3 \end{cases}$$

Answer: $y = -5$

$x = 3$

$z = 0$

12. $4x - y + 5z = 4$ Equation 1
$2x + y - z = 0$ Equation 2
$2x + 4y + 8z = 0$ Equation 3

$$\begin{cases} 4x - y + 5z = 4 & \\ -3y + 7z = 4 & \text{Eq.1} - 2\text{Eq.2} \\ 3y + 9z = 0 & -\text{Eq.2} + \text{Eq.3} \end{cases}$$

$$\begin{cases} 4x - y + 5z = 4 & \\ -3y + 7z = 4 & \\ 16z = 4 & \text{Eq.2} + \text{Eq.3} \end{cases}$$

Answer: $z = \frac{1}{4}$

$y = -\frac{3}{4}$

$x = \frac{1}{2}$

13. $\begin{cases} 3x + 2y - z = 5 \implies 6x + 4y - 2z = 10 \\ 6x - y + 5z = 2 \implies -6x + y - 5z = -2 \end{cases}$

$5y - 7z = 8$

$$y = \frac{8 + 7z}{5}$$

$3x + 2y - z = 5$
$12x - 2y + 10z = 4$
$15x + 9z = 9$

$$x = \frac{9 - 9z}{15} = \frac{3 - 3z}{5}$$

Let $z = a$, then $x = \dfrac{3 - 3a}{5}$ and $y = \dfrac{8 + 7a}{5}$.

14. $y = ax^2 + bx + c$ passes through $(0, -1)$, $(1, 4)$, and $(2, 13)$.

At $(0, -1)$: $-1 = a(0)^2 + b(0) + c \implies c = -1$

At $(1, 4)$: $4 = a(1)^2 + b(1) - 1 \implies 5 = a + b \implies 5 = a + b$

At $(2, 13)$: $13 = a(2)^2 + b(2) - 1 \implies 14 = 4a + 2b \implies -7 = -2a - b$

$-2 = -a$

$a = 2$

$b = 3$

Thus, $y = 2x^2 + 3x - 1$.

15. $s = \frac{1}{2}at^2 + v_0t + s_0$ passes through $(1, 12)$, $(2, 5)$, and $(3, 4)$.

At $(1, 12)$: $12 = \frac{1}{2}a + v_0 + s_0 \implies$
At $(2, 5)$: $5 = 2a + 2v_0 + s_0 \implies$
At $(3, 4)$: $4 = \frac{9}{2}a + 3v_0 + s_0 \implies$

$$\begin{cases} \frac{1}{2}a + v_0 + s_0 = 12 & \\ -a + s_0 = 19 & 2\text{Eq.1} - \text{Eq.2} \\ -3a + s_0 = 7 & 3\text{Eq.2} - 2\text{Eq.3} \end{cases}$$

$$\begin{cases} \frac{1}{2}a + v_0 + s_0 = 12 & \\ -a + s_0 = 19 & \\ -2a = -12 & -\text{Eq.2} + \text{Eq.3} \end{cases}$$

$a = 6$

$s_0 = 25$

$v_0 = -16$

Thus,

$s = \frac{1}{2}(6)t^2 - 16t + 25 = 3t^2 - 16t + 25$.

16.
$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & -5 & 9 \end{bmatrix}$$

$$-3R_1 + R_2 \rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \end{bmatrix}$$

$$2R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \end{bmatrix}$$

17. $3x + 5y = 3$

$2x - y = -11$

$$\begin{bmatrix} 3 & 5 & \vdots & 3 \\ 2 & -1 & \vdots & -11 \end{bmatrix}$$

$$\begin{array}{r} -R_2 + R_1 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 2 & -1 & \vdots & -11 \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \end{array} \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 0 & -13 & \vdots & -39 \end{bmatrix}$$

$$\begin{array}{r} \\ -\frac{1}{13}R_2 \rightarrow \end{array} \begin{bmatrix} 1 & 6 & \vdots & 14 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$$

$$\begin{array}{r} -6R_2 + R_1 \rightarrow \\ \\ \end{array} \begin{bmatrix} 1 & 0 & \vdots & -4 \\ 0 & 1 & \vdots & 3 \end{bmatrix}$$

Answer: $x = -4, y = 3$

18. $\begin{cases} 2x + 3y = -3 \\ 3x + 2y = 8 \\ x + y = 1 \end{cases}$

$$\begin{bmatrix} 2 & 3 & \vdots & -3 \\ 3 & 2 & \vdots & 8 \\ 1 & 1 & \vdots & 1 \end{bmatrix}$$

$$\begin{array}{r} R_3 \\ \\ R_1 \end{array} \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 3 & 2 & \vdots & 8 \\ 2 & 3 & \vdots & -3 \end{bmatrix}$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -2R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 1 & \vdots & 1 \\ 0 & -1 & \vdots & 5 \\ 0 & 1 & \vdots & -5 \end{bmatrix}$$

$$\begin{array}{r} R_2 + R_1 \rightarrow \\ -R_2 \rightarrow \\ -R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & \vdots & 6 \\ 0 & 1 & \vdots & -5 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

Answer: $x = 6, y = -5$

19. $\begin{cases} x + 3z = -5 \\ 2x + y = 0 \\ 3x + y - z = 3 \end{cases}$

$$\begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 2 & 1 & 0 & \vdots & 0 \\ 3 & 1 & -1 & \vdots & 3 \end{bmatrix}$$

$$\begin{array}{r} \\ -2R_1 + R_2 \rightarrow \\ -3R_1 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 0 & 1 & -6 & \vdots & 10 \\ 0 & 1 & -10 & \vdots & 18 \end{bmatrix}$$

$$\begin{array}{r} \\ \\ -R_2 + R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & 3 & \vdots & -5 \\ 0 & 1 & -6 & \vdots & 10 \\ 0 & 0 & -4 & \vdots & 8 \end{bmatrix}$$

$$\begin{array}{r} -3R_3 + R_1 \rightarrow \\ 6R_3 + R_2 \rightarrow \\ -\frac{1}{4}R_3 \rightarrow \end{array} \begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -2 \\ 0 & 0 & 1 & \vdots & -2 \end{bmatrix}$$

Answer: $x = 1, y = -2, z = -2$

20. $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & -7 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -4 & -12 \\ 5 & 6 \end{bmatrix}$

21. $3A - 5B = 3\begin{bmatrix} 9 & 1 \\ -4 & 8 \end{bmatrix} - 5\begin{bmatrix} 6 & -2 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} -3 & 13 \\ -27 & -1 \end{bmatrix}$

22. $f(A) = \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}^2 - 7\begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix} + 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 7 & 1 \end{bmatrix} - \begin{bmatrix} 21 & 0 \\ 49 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ 28 & 1 \end{bmatrix} - \begin{bmatrix} 21 & 0 \\ 49 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 \\ -21 & 2 \end{bmatrix}$

23. False

$$(A + B)(A + 3B) = A(A + 3B) + B(A + 3B)$$
$$= A^2 + 3AB + BA + 3B^2$$

24. $$\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$$

$$\begin{array}{r} 2R_2 + R_1 \rightarrow \\ -R_2 \rightarrow \end{array}\left[\begin{array}{cc:cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

25. $$\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 6 & 5 & 0 & 1 & 0 \\ 6 & 10 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \\ -6R_1 + R_3 \rightarrow \end{array}\left[\begin{array}{ccc:ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 2 & -3 & 1 & 0 \\ 0 & 4 & 2 & -6 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} -\frac{1}{3}R_2 + R_1 \rightarrow \\ \frac{1}{3}R_2 \rightarrow \\ -4R_2 + R_3 \rightarrow \end{array}\left[\begin{array}{ccc:ccc} 1 & 0 & \frac{1}{3} & 2 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{2}{3} & -1 & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{2}{3} & -2 & -\frac{4}{3} & 1 \end{array}\right]$$

$$\begin{array}{r} \frac{1}{2}R_3 + R_1 \rightarrow \\ R_3 + R_2 \rightarrow \\ -\frac{3}{2}R_3 \rightarrow \end{array}\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & \frac{1}{2} \\ 0 & 1 & 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & 3 & 2 & -\frac{3}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 1 & -1 & \frac{1}{2} \\ -3 & -1 & 1 \\ 3 & 2 & -\frac{3}{2} \end{bmatrix}$$

26. (a) $x + 2y = 4$

$3x + 5y = 1$

$$\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$$\begin{array}{r} \\ -3R_1 + R_2 \rightarrow \end{array}\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$$

$$\begin{array}{r} -2R_2 + R_1 \rightarrow \\ -R_2 \rightarrow \end{array}\left[\begin{array}{cc:cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$X = A^{-1}B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -18 \\ 11 \end{bmatrix}$$

$x = -18, y = 11$

(b) $x + 2y = 3$

$3x + 5y = -2$

$$X = A^{-1}B = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -19 \\ 11 \end{bmatrix}$$

$x = -19, y = 11$

27. $$\begin{vmatrix} 6 & -1 \\ 3 & 4 \end{vmatrix} = 24 - (-3) = 27$$

28. $$\begin{vmatrix} 1 & 3 & -1 \\ 5 & 9 & 0 \\ 6 & 2 & -5 \end{vmatrix} = 1(-45) + (-3)(-25) + (-1)(-44)$$
$$= 74$$

29. $$\begin{vmatrix} 1 & 4 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 3 & 5 & -1 & 1 \\ 2 & 0 & 6 & 1 \end{vmatrix} = -7$$

30. $$\begin{vmatrix} 6 & 4 & 3 & 0 & 6 \\ 0 & 5 & 1 & 4 & 8 \\ 0 & 0 & 2 & 7 & 3 \\ 0 & 0 & 0 & 9 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 6(5)(2)(9)(1) = 540$$

31. $$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 7 & 1 \\ 5 & 0 & 1 \\ 3 & 9 & 1 \end{vmatrix} = \frac{1}{2}(31)$$
$$= 15.5 \text{ square units}$$

32. $$\begin{vmatrix} x & y & 1 \\ 2 & 7 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 3x - 3y + 15 = 0$$

or $\quad x - y + 5 = 0$

33. $x = \dfrac{\begin{vmatrix} 4 & -7 \\ 11 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -7 \\ 2 & 5 \end{vmatrix}} = \dfrac{97}{44}$

34. $z = \dfrac{\begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & -1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 0 & 1 \\ 0 & 1 & 4 \\ 1 & -1 & 0 \end{vmatrix}} = \dfrac{14}{11}$

35. $y = \dfrac{\begin{vmatrix} 721.4 & 33.77 \\ 45.9 & 19.85 \end{vmatrix}}{\begin{vmatrix} 721.4 & -29.1 \\ 45.9 & 105.6 \end{vmatrix}}$

$= \dfrac{12{,}769.747}{77{,}515.530} \approx 0.1647$

Chapter 8 Practice Test Solutions

1. $a_n = \dfrac{2n}{(n+2)!}$

$a_1 = \dfrac{2(1)}{3!} = \dfrac{2}{6} = \dfrac{1}{3}$

$a_2 = \dfrac{2(2)}{4!} = \dfrac{4}{24} = \dfrac{1}{6}$

$a_3 = \dfrac{2(3)}{5!} = \dfrac{6}{120} = \dfrac{1}{20}$

$a_4 = \dfrac{2(4)}{6!} = \dfrac{8}{720} = \dfrac{1}{90}$

$a_5 = \dfrac{2(5)}{7!} = \dfrac{10}{5040} = \dfrac{1}{504}$

Terms: $\dfrac{1}{3}, \dfrac{1}{6}, \dfrac{1}{20}, \dfrac{1}{90}, \dfrac{1}{504}$

2. $a_n = \dfrac{n+3}{3^n}$

3. $\displaystyle\sum_{i=1}^{6}(2i-1) = 1 + 3 + 5 + 7 + 9 + 11 = 36$

4. $a_1 = 23,\ d = -2$

$a_2 = a_1 + d = 21$

$a_3 = a_2 + d = 19$

$a_4 = a_3 + d = 17$

$a_5 = a_4 + d = 15$

Terms: 23, 21, 19, 17, 15

5. $a_1 = 12, d = 3, n = 50$

$a_n = a_1 + (n-1)d$

$a_{50} = 12 + (50-1)3 = 159$

6. $a_1 = 1$

$a_{200} = 200$

$S_n = \dfrac{n}{2}(a_1 + a_n)$

$S_{200} = \dfrac{200}{2}(1 + 200) = 20{,}100$

7. $a_1 = 7, r = 2$

$a_2 = a_1 r = 14$

$a_3 = a_1 r^2 = 28$

$a_4 = a_1 r^3 = 56$

$a_5 = a_1 r^4 = 112$

Terms: 7, 14, 28, 56, 112

8. $\sum_{n=0}^{9} 6\left(\frac{2}{3}\right)^n, a_1 = 6, r = \frac{2}{3}, n = 10$

$$S_n = \frac{a_1(1 - r^n)}{1 - r} = \frac{6\left(1 - (2/3)^{10}\right)}{1 - (2/3)} \approx 17.6879$$

9. $\sum_{n=0}^{\infty} (0.03)^n, a_1 = 1, r = 0.03$

$$S = \frac{a_1}{1 - r} = \frac{1}{1 - 0.03} = \frac{1}{0.97} = \frac{100}{97} \approx 1.0309$$

10. For $n = 1, 1 = \frac{1(1+1)}{2}$. Assume that $1 + 2 + 3 = 4 + \cdot \cdot \cdot + k = \frac{k(k+1)}{2}$. Now for $n = k + 1$,

$$\begin{aligned} 1 + 2 + 3 + 4 + \cdot \cdot \cdot + k + (k + 1) &= \frac{k(k+1)}{2} + k + 1 \\ &= \frac{k(k+1)}{2} + \frac{2(k+1)}{2} \\ &= \frac{(k+1)(k+2)}{2}. \end{aligned}$$

Thus, $1 + 2 + 3 + 4 + \cdot \cdot \cdot + n = \frac{n(n+1)}{2}$ for all integers $n \geq 1$.

11. For $n = 4, 4! > 2^4$. Assume that $k! > 2^k$. Then

$$\begin{aligned} (k + 1)! = (k + 1)(k!) &> (k + 1)2^k > 2 \cdot 2^k \\ &= 2^{k+1}. \end{aligned}$$

Thus, $n! > 2^n$ for all integers $n \geq 4$.

12. ${}_{13}C_4 = \frac{13!}{(13 - 4)!4!} = 715$

13.
$$\begin{aligned} (x + 3)^5 &= x^5 + 5x^4(3) + 10x^3(3)^2 + 10x^2(3)^3 + 5x(3)^4 + (3)^5 \\ &= x^5 + 15x^4 + 90x^3 + 270x^2 + 405x + 243 \end{aligned}$$

14. ${}_{12}C_5 x^7(-2)^5 = -25{,}344x^7$

15. ${}_{30}P_4 = \frac{30!}{(30 - 4)!} = 657{,}720$

16. $6! = 720$ ways

17. ${}_{12}P_3 = 1320$

18.
$$\begin{aligned} P(2) + P(3) + P(4) &= \tfrac{1}{36} + \tfrac{2}{36} + \tfrac{3}{36} \\ &= \tfrac{6}{36} = \tfrac{1}{6} \end{aligned}$$

19. $P(K, B10) = \frac{4}{52} \cdot \frac{2}{51} = \frac{2}{663}$

20. Let A = probability of no faulty units.

$P(A) = \left(\frac{997}{1000}\right)^{50} \approx 0.8605$

$P(A') = 1 - P(A) \approx 0.1395$

Chapter 9 Practice Test Solutions

1. $x^2 - 6x - 4y + 1 = 0$

$$x^2 - 6x + 9 = 4y - 1 + 9$$

$$(x - 3)^2 = 4y + 8$$

$$(x - 3)^2 = 4(1)(y + 2) \implies p = 1$$

Vertex: $(3, -2)$

Focus: $(3, -1)$

Directrix: $y = -3$

2. Vertex: $(2, -5)$

Focus: $(2, -6)$

Vertical axis; opens downward with $p = -1$

$$(x - h)^2 = 4p(y - k)$$

$$(x - 2)^2 = 4(-1)(y + 5)$$

$$x^2 - 4x + 4 = -4y - 20$$

$$x^2 - 4x + 4y + 24 = 0$$

3. $x^2 + 4y^2 - 2x + 32y + 61 = 0$

$$(x^2 - 2x + 1) + 4(y^2 + 8y + 16) = -61 + 1 + 64$$

$$(x - 1)^2 + 4(y + 4)^2 = 4$$

$$\frac{(x - 1)^2}{4} + \frac{(y + 4)^2}{1} = 1$$

$a = 2, b = 1, c = \sqrt{3}$

Horizontal major axis

Center: $(1, -4)$

Foci: $\left(1 \pm \sqrt{3}, -4\right)$

Vertices: $(3, -4), (-1, -4)$

Eccentricity: $e = \dfrac{\sqrt{3}}{2}$

4. Vertices: $(0, \pm 6)$

Eccentricity: $e = \dfrac{1}{2}$

Center: $(0, 0)$

Vertical major axis

$$a = 6, e = \frac{c}{a} = \frac{c}{6} = \frac{1}{2} \implies c = 3$$

$$b^2 = (6)^2 - (3)^2 = 27$$

$$\frac{x^2}{27} + \frac{y^2}{36} = 1$$

5. $16y^2 - x^2 - 6x - 128y + 231 = 0$

$$16(y^2 - 8y + 16) - (x^2 + 6x + 9) = -231 + 256 - 9$$

$$16(y - 4)^2 - (x + 3)^2 = 16$$

$$\frac{(y - 4)^2}{1} - \frac{(x + 3)^2}{16} = 1$$

$a = 1, b = 4, c = \sqrt{17}$

Center: $(-3, 4)$

Vertical transverse axis

Vertices: $(-3, 5), (-3, 3)$

Foci: $\left(-3, 4 \pm \sqrt{17}\right)$

Asymptotes: $y = 4 \pm \dfrac{1}{4}(x + 3)$

6. Vertices: $(\pm 3, 2)$

Foci: $(\pm 5, 2)$

Center: $(0, 2)$

Horizontal transverse axis

$a = 3, c = 5, b = 4$

$$\frac{(x - 0)^2}{9} - \frac{(y - 2)^2}{16} = 1$$

$$\frac{x^2}{9} - \frac{(y - 2)^2}{16} = 1$$

7. $5x^2 + 2xy + 5y^2 - 10 = 0$

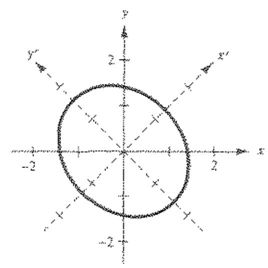
$A = 5, B = 2, C = 5$

$\cot 2\theta = \dfrac{5-5}{2} = 0$

$2\theta = \dfrac{\pi}{2} \Rightarrow \theta = \dfrac{\pi}{4}$

$x = x' \cos \dfrac{\pi}{4} - y' \sin \dfrac{\pi}{4} = \dfrac{x' - y'}{\sqrt{2}}$

$y = x' \sin \dfrac{\pi}{4} + y' \cos \dfrac{\pi}{4} = \dfrac{x' + y'}{\sqrt{2}}$



$$5\left(\frac{x'-y'}{\sqrt{2}}\right)^2 + 2\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) + 5\left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 10 = 0$$

$$\frac{5(x')^2}{2} - \frac{10x'y'}{2} + \frac{5(y')^2}{2} + (x')^2 - (y')^2 + \frac{5(x')^2}{2} + \frac{10x'y'}{2} + \frac{5(y')^2}{2} - 10 = 0$$

$$6(x')^2 + 4(y')^2 - 10 = 0$$

$$\frac{3(x')^2}{5} + \frac{2(y')^2}{5} = 1$$

$$\frac{(x')^2}{5/3} + \frac{(y')^2}{5/2} = 1$$

Ellipse centered at the origin

8. (a) $6x^2 - 2xy + y^2 = 0$

$A = 6, B = -2, C = 1$

$B^2 - 4AC = (-2)^2 - 4(6)(1) = -20 < 0$

Ellipse

(b) $x^2 + 4xy + 4y^2 - x - y + 17 = 0$

$A = 1, B = 4, C = 4$

$B^2 - 4AC = (4)^2 - 4(1)(4) = 0$

Parabola

9. $x = 3 - 2\sin\theta, y = 1 + 5\cos\theta$

$\dfrac{x-3}{-2} = \sin\theta, \dfrac{y-1}{5} = \cos\theta$

$\left(\dfrac{x-3}{-2}\right)^2 + \left(\dfrac{y-1}{5}\right)^2 = 1$

$\dfrac{(x-3)^2}{4} + \dfrac{(y-1)^2}{25} = 1$

10. $x = e^{2t}, y = e^{4t}$

$x > 0, y > 0$

$y = (e^{2t})^2 = (x)^2 = x^2, x > 0, y > 0$

11. Polar: $\left(\sqrt{2}, \dfrac{3\pi}{4}\right)$

$x = \sqrt{2}\cos\dfrac{3\pi}{4} = \sqrt{2}\left(-\dfrac{1}{\sqrt{2}}\right) = -1$

$y = \sqrt{2}\sin\dfrac{3\pi}{4} = \sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right) = 1$

Rectangular: $(-1, 1)$

12. Rectangular: $(\sqrt{3}, -1)$

$r = \pm\sqrt{(\sqrt{3})^2 + (-1)^2} = \pm 2$

$\tan\theta = \dfrac{-1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

$\theta = \dfrac{5\pi}{6}$ or $\theta = \dfrac{11\pi}{6}$

Polar: $\left(-2, \dfrac{5\pi}{6}\right)$ or $\left(2, \dfrac{11\pi}{6}\right)$

13. Rectangular: $4x - 3y = 12$

Polar: $4r\cos\theta - 3r\sin\theta = 12$

$$r(4\cos\theta - 3\sin\theta) = 12$$

$$r = \frac{12}{4\cos\theta - 3\sin\theta}$$

14. Polar: $r = 5\cos\theta$

$$r^2 = 5r\cos\theta$$

Rectangular: $x^2 + y^2 = 5x$

$$x^2 + y^2 - 5x = 0$$

15. $r = 1 - \cos\theta$

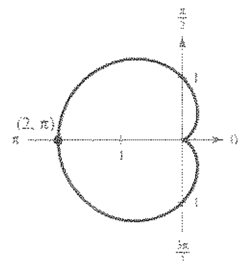
Cardioid

Symmetry: Polar axis

Maximum value of $|r|$: $r = 2$ when $\theta = \pi$.

Zero of r: $r = 0$ when $\theta = 0$.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	0	1	2	1



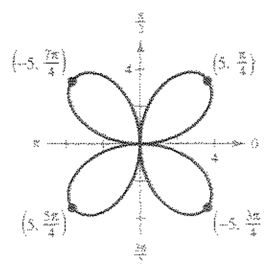
16. $r = 5\sin 2\theta$

Rose curve with four petals

Symmetry: Polar axis, $\theta = \frac{\pi}{2}$, and pole

Maximum value of $|r|$: $|r| = 5$ when $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

Zeros of r: $r = 0$ when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$.

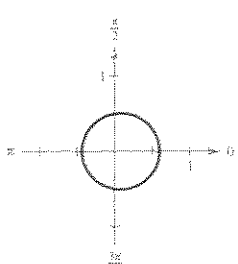


17. $r = \dfrac{3}{6 - \cos\theta}$

$$r = \frac{1/2}{1 - (1/6)\cos\theta}$$

$e = \frac{1}{6} < 1$, so the graph is an ellipse.

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
r	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{1}{2}$



18. Parabola

Vertex: $\left(6, \frac{\pi}{2}\right)$

Focus: $(0, 0)$

$$e = 1$$

$$r = \frac{ep}{1 + e\sin\theta}$$

$$r = \frac{p}{1 + \sin\theta}$$

$$6 = \frac{p}{1 + \sin(\pi/2)}$$

$$6 = \frac{p}{2}$$

$$12 = p$$

$$r = \frac{12}{1 + \sin\theta}$$

Chapter 10 Practice Test Solutions

1. Let $A = (0, 0, 0)$, $B = (1, 2, -4)$, $C = (0, -2, -1)$.

Side AB: $\sqrt{1^2 + 2^2 + 4^2} = \sqrt{21}$

Side AC: $\sqrt{0^2 + 2^2 + 1^2} = \sqrt{5}$

Side BC: $\sqrt{(-1)^2 + (-2 - 2)^2 + (-1 + 4)^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$

$BC^2 = AB^2 + AC^2$

$26 = 21 + 5$

2. $(x - 0)^2 + (y - 4)^2 + (z - 1)^2 = 5^2$

$x^2 + (y - 4)^2 + (z - 1)^2 = 25$

3. $(x^2 + 2x + 1) + y^2 + (z^2 - 4z + 4) = 1 + 4 + 11$

$(x + 1)^2 + y^2 + (z - 2)^2 = 16$

Center: $(-1, 0, 2)$

Radius: 4

4. $\mathbf{u} - 3\mathbf{v} = \langle 1, 0, -1\rangle - 3\langle 4, 3, -6\rangle$

$= \langle 1, 0, -1\rangle - \langle 12, 9, -18\rangle$

$= \langle -11, -9, 17\rangle$

5. $\frac{1}{2}\mathbf{v} = \frac{1}{2}\langle 2, 4, -6\rangle = \langle 1, 2, -3\rangle$

$\left\|\frac{1}{2}\mathbf{v}\right\| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

6. $\mathbf{u} \cdot \mathbf{v} = \langle 2, 1, -3\rangle \cdot \langle 1, 1, -2\rangle$

$= 2 + 1 + 6 = 9$

7. Because $\mathbf{v} = \langle -3, -3, 3\rangle = -3\langle 1, 1, -1\rangle = -3\mathbf{u}$, $\mathbf{u}$ and $\mathbf{v}$ are parallel.

8. $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 0 & 2 \\ 1 & -1 & 3 \end{vmatrix} = \langle 2, 5, 1\rangle$

$\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v}) = \langle -2, -5, -1\rangle$

9. $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1 & 0 & 4 \end{vmatrix}$

$= 1(-4) - 1(-1) + 1(1)$

$= -4 + 1 + 1 = -2$

Volume $= |\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})| = |-2| = 2$

10. $\mathbf{v} = \langle (2 - 0), -3 - (-3), 4 - 3\rangle = \langle 2, 0, 1\rangle$

$x = 2 + 2t$, $y = -3$, $z = 4 + t$

11. $1(x - 1) - 1(y - 2) + 0(z - 3) = 0$

$x - 1 - y + 2 = 0$

$x - y + 1 = 0$

12. $\overrightarrow{AB} = \langle 1, 1, 1\rangle$, $\overrightarrow{AC} = \langle 1, 2, 3\rangle$

$\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 1, -2, 1\rangle$

Plane: $1(x - 0) - 2(y - 0) + (z - 0) = 0$

$x - 2y + z = 0$

13. $\mathbf{n}_1 = \langle 1, 1, -1\rangle$, $\mathbf{n}_2 = \langle 3, -4, -1\rangle$

$\mathbf{n}_1 \cdot \mathbf{n}_2 = 3 - 4 + 1 = 0 \Rightarrow$ Orthogonal planes

14. $\mathbf{n} = \langle 1, 2, 1\rangle$, $Q = (1, 1, 1)$, $P = (0, 0, 6)$ on plane, $\overrightarrow{PQ} = \langle 1, 1, -5\rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|1 + 2 - 5|}{\sqrt{1 + 4 + 1}} = \frac{2}{\sqrt{6}} = \frac{\sqrt{6}}{3}$$

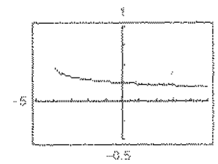
Chapter 11 Practice Test Solutions

1.

x	2.9	2.99	3	3.01	3.1
$f(x)$	0.1695	0.1669	?	0.1664	0.1639

$$\lim_{x\to 3} \frac{x-3}{x^2-9} \approx 0.1667$$

2. $\displaystyle \lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \approx \frac{1}{4}$

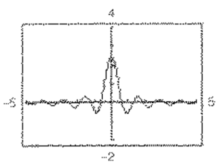


3. $\displaystyle \lim_{x\to 2} e^{x-2} = e^{2-2} = e^0 = 1$

4. $\displaystyle \lim_{x\to 1} \frac{x^3-1}{x-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{x-1}$

$\displaystyle = \lim_{x\to 1} (x^2+x+1) = 3$

5. $\displaystyle \lim_{x\to 0} \frac{\sin 5x}{2x} \approx 2.5$



6. The limit does not exist. If

$$f(x) = \frac{|x+2|}{x+2},$$

then $f(x) = 1$ for $x > -2$, and $f(x) = -1$ for $x < -2$.

7.
$$\begin{aligned} m_{\text{sec}} &= \frac{f(4+h)-f(4)}{h} \\ &= \frac{\sqrt{4+h}-2}{h} \\ &= \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2} \\ &= \frac{(4+h)-4}{h\left[\sqrt{4+h}+2\right]} \\ &= \frac{h}{h\left[\sqrt{4+h}+2\right]} \\ &= \frac{1}{\sqrt{4+h}+2},\ h \neq 0 \end{aligned}$$

$$m = \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

8.
$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0} \frac{[3(x+h)-1]-[3x-1]}{h} \\ &= \lim_{h\to 0} \frac{3x+3h-1-3x+1}{h} \\ &= \lim_{h\to 0} \frac{3h}{h} = \lim_{h\to 0} 3 = 3 \end{aligned}$$

9. (a) $\displaystyle \lim_{x\to\infty} \frac{3}{x^4} = 0$

(b) $\displaystyle \lim_{x\to -\infty} \frac{x^2}{x^2+3} = 1$

(c) $\displaystyle \lim_{x\to\infty} \frac{|x|}{1-x} = -1$

10. $\displaystyle a_1 = 0,\ a_2 = \frac{1-4}{8+1} = -\frac{1}{3},\ a_3 = \frac{1-9}{18+1} = -\frac{8}{19},$

$\displaystyle a_4 = \frac{1-16}{33} = -\frac{15}{33}$

$\displaystyle \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1-n^2}{2n^2+1} = -\frac{1}{2}$

11. $\displaystyle \sum_{i=1}^{25} i^2 + \sum_{i=1}^{25} i = \frac{25(26)(51)}{6} + \frac{25(26)}{2} = \frac{25(26)}{6}[51+3] = \frac{25(26)(54)}{6} = 5850$

12. $\displaystyle\sum_{i=1}^{n} \frac{i^2}{n^3} = \frac{1}{n^3}\sum_{i=1}^{n} i^2 = \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] = \frac{2n^2+3n+1}{6n^2} = S(n)$

$\displaystyle\lim_{n\to\infty} S(n) = \frac{1}{3}$

13. Width of rectangles: $\dfrac{b-a}{n} = \dfrac{1}{n}$

Height: $f\left(a + \dfrac{(b-a)i}{n}\right) = f\left(\dfrac{i}{n}\right) = 1 - \left(\dfrac{i}{n}\right)^2$

$\displaystyle A_n \approx \sum_{i=1}^{n}\left[1 - \frac{i^2}{n^2}\right]\frac{1}{n} = \sum_{i=1}^{n}\frac{1}{n} - \sum_{i=1}^{n}\frac{i^2}{n^3} = 1 - \frac{1}{n^2}\,\frac{n(n+1)(2n+1)}{6}$

$\displaystyle A = \lim_{n\to\infty} A_n = 1 - \frac{1}{3} = \frac{2}{3}$

PART II

Chapter 1 Chapter Test Solutions

1. $5x + 2y = 3$

$$2y = -5x + 3$$

$$y = -\frac{5}{2}x + \frac{3}{2}$$

Slope $= -\frac{5}{2}$

(a) Parallel line

$$y - 4 = -\frac{5}{2}(x - 0)$$

$$y = -\frac{5}{2}x + 4$$

$$5x + 2y - 8 = 0$$

(b) Perpendicular line

$$y - 4 = \frac{2}{5}(x - 0)$$

$$y = \frac{2}{5}x + 4$$

$$2x - 5y + 20 = 0$$

2. Slope $= \dfrac{4 - (-1)}{-3 - 2} = \dfrac{5}{-5} = -1$

$$y + 1 = -1(x - 2)$$

$$y = -x + 1$$

3. No, for some x there corresponds more than one value of y. For instance, if $x = 1$, $y = \pm 1/\sqrt{3}$.

4. $f(x) = |x + 2| - 15$

(a) $f(-8) = |-8 + 2| - 15 = 6 - 15 = -9$

(b) $f(14) = |14 + 2| - 15 = 16 - 15 = 1$

(c) $f(t - 6) = |t - 6 + 2| - 15 = |t - 4| - 15$

5. $3 - x \geq 0 \implies$ domain is all $x \leq 3$.

6. $C = 5.60x + 24{,}000$

$$P = R - C$$

$$= 99.50x - (5.60x + 24{,}000)$$

$$= 93.9x - 24{,}000$$

7. $f(-x) = 2(-x)^3 - 3(-x)$

$$= -2x^3 + 3x = -f(x)$$

Odd

8. $f(-x) = 3(-x)^4 + 5(-x)^2$

$$= 3x^4 + 5x^2 = f(x)$$

Even

9. $h(x) = \frac{1}{4}x^4 - 2x^2 = \frac{1}{4}x^2(x^2 - 8)$

By graphing h, you see that the graph is increasing on $(-2, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(0, 2)$.

10. $g(t) = |t + 2| - |t - 2|$

By graphing g, you see that the graph is increasing on $(-2, 2)$, and constant on $(-\infty, -2)$ and $(2, \infty)$.

11. Relative minimum: $(-3.33, -6.52)$

Relative maximum: $(0, 12)$

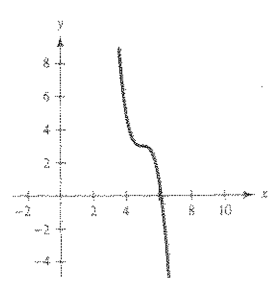
12. Relative minimum: $(0.77, 1.81)$

Relative maximum: $(-0.77, 2.19)$

13. (a) Parent function $f(x) = x^3$

(b) g is obtained from f by a horizontal shift five units to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift three units upward.

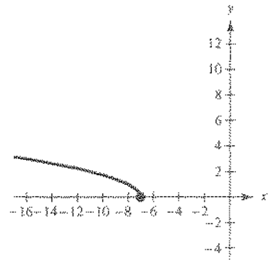
(c)



14. (a) Parent function $f(x) = \sqrt{x}$

(b) g is obtained from f by a reflection in the y-axis, and a horizontal shift seven units to the left.

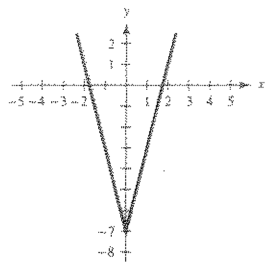
(c)



15. (a) Parent function $f(x) = |x|$

(b) $g(x) = 4|-x| - 7 = 4|x| - 7$ is obtained from f by a vertical stretch of 4 followed by a vertical shift seven units downward.

(c)



16. (a) $(f - g)(x) = x^2 - \sqrt{2 - x}$

Domain: $x \le 2$

(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2}{\sqrt{2 - x}}$

Domain: $x < 2$

(c) $(f \circ g)(x) = f\left(\sqrt{2 - x}\right) = 2 - x$

Domain: $x \le 2$

(d) $(g \circ f)x = g(x^2) = \sqrt{2 - x^2}$

Domain: $-\sqrt{2} \le x \le \sqrt{2}$

17. $f(x) = x^3 + 8$

Yes, f is one-to-one and has an inverse function.

$$y = x^3 + 8$$
$$x = y^3 + 8$$
$$x - 8 = y^3$$
$$\sqrt[3]{x - 8} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 8}$$

18. $f(x) = x^2 + 6$

No, f is not one-to-one, and does not have an inverse function.

19. $f(x) = \dfrac{3x\sqrt{x}}{8}$

Yes, f is one-to-one and has an inverse function.

$$y = \tfrac{3}{8}x^{3/2},\ x \ge 0, y \ge 0$$
$$x = \tfrac{3}{8}y^{3/2},\ y \ge 0,\ x \ge 0$$
$$\tfrac{8}{3}x = y^{3/2}$$
$$\left(\tfrac{8}{3}x\right)^{2/3} = y$$
$$f^{-1}(x) = \left(\tfrac{8}{3}x\right)^{2/3},\quad x \ge 0$$

20. $y = 18.30t - 76.2$, $r \approx 0.99622$

$y = 200$ for $t \approx 15$, or 2005

Chapter 2 Chapter Test Solutions

1. $y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x + 2)^2 - 1$

Vertex: $(-2, -1)$

$x = 0 \implies y = 3$

$y = 0 \implies x^2 + 4x + 3 = 0 \implies (x + 3)(x + 1) = 0 \implies x = -1, -3$

Intercepts: $(0, 3), (-1, 0), (-3, 0)$

2. Let $y = a(x-h)^2 + k$. The vertex $(3, -6)$ implies that $y = a(x-3)^2 - 6$. For $(0, 3)$ you obtain

$3 = a(0-3)^2 - 6 = 9a - 6 \implies a = 1.$

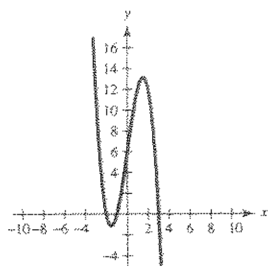
Thus, $y = (x-3)^2 - 6 = x^2 - 6x + 3$.

3. $f(x) = 4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x+1)^2$

Zeros: 0 (multiplicity 1)

$-\frac{1}{2}$ (multiplicity 2)

4. $f(x) = -x^3 + 7x + 6$



5.
$$\begin{array}{r}
3x \\
x^2+1 \overline{\smash{)}\,3x^3 + 0x^2 + 4x - 1} \\
\underline{3x^3 + 3x } \\
x - 1
\end{array}$$

$3x + \dfrac{x-1}{x^2+1}$

6.
$$\begin{array}{r|rrrrr}
2 & 2 & 0 & -5 & 0 & -3 \\
  &   & 4 & 8 & 6 & 12 \\ \hline
  & 2 & 4 & 3 & 6 & 9
\end{array}$$

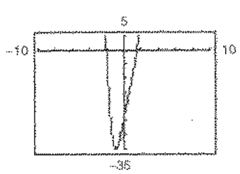
$2x^3 + 4x^2 + 3x + 6 + \dfrac{9}{x-2}$

7.
$$\begin{array}{r|rrrrr}
-2 & 3 & 0 & -6 & 5 & -1 \\
   &   & -6 & 12 & -12 & 14 \\ \hline
   & 3 & -6 & 6 & -7 & 13
\end{array}$$

$f(-2) = 13$

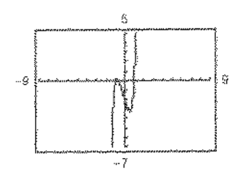
8. Possible rational zeros:

$\pm 24, \pm 12, \pm 8, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1, \pm\frac{3}{2}, \pm\frac{1}{2}$



Rational zeros: $-2, \frac{3}{2}$

9. Possible rational zeros: $\pm 2, \pm 1, \pm\frac{2}{3}, \pm\frac{1}{3}$



Rational zeros: $\pm 1, -\frac{2}{3}$

10. $f(x) = x^3 - 7x^2 + 11x + 19$

$= (x+1)(x^2 - 8x + 19)$

For the quadratic,

$x = \dfrac{8 \pm \sqrt{64 - 4(19)}}{2} = 4 \pm \sqrt{3}i.$

Zeros: $-1, 4 \pm \sqrt{3}i$

$f(x) = (x+1)(x - 4 + \sqrt{3}i)(x - 4 - \sqrt{3}i)$

11. $(-8 - 3i) + (-1 - 15i) = -9 - 18i$

12. $\left(10 + \sqrt{-20}\right) - \left(4 - \sqrt{-14}\right) = 6 + 2\sqrt{5}i + \sqrt{14}i = 6 + \left(2\sqrt{5} + \sqrt{14}\right)i$

13. $(2 + i)(6 - i) = 12 + 6i - 2i + 1 = 13 + 4i$

14. $(4 + 3i)^2 - (5 + i)^2 = (16 + 24i - 9) - (25 + 10i - 1) = -17 + 14i$

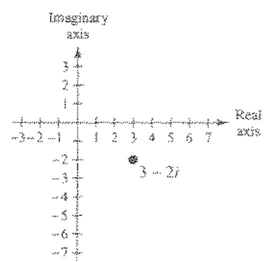
15. $\dfrac{8 + 5i}{6 - i} \cdot \dfrac{6 + i}{6 + i} = \dfrac{48 + 30i + 8i - 5}{36 + 1} = \dfrac{43}{37} + \dfrac{38}{37}i$

16. $\dfrac{5i}{2 + i} \cdot \dfrac{2 - i}{2 - i} = \dfrac{10i + 5}{4 + 1} = 1 + 2i$

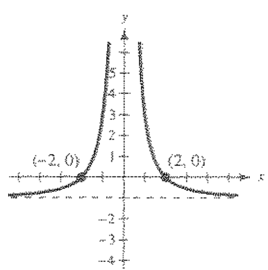
17. $\dfrac{(2i - 1)}{(3i + 2)} \cdot \dfrac{2 - 3i}{2 - 3i} = \dfrac{6 - 2 + 4i + 3i}{4 + 9}$

$= \dfrac{4}{13} + \dfrac{7}{13}i$

18.



19.



Vertical asymptote: $x = 0$

Intercepts: $(2, 0), (-2, 0)$

Symmetry: y-axis

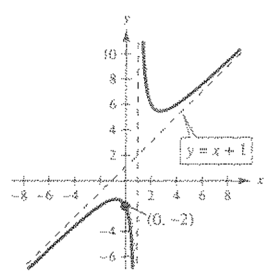
Horizontal asymptote: $y = -1$

20. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

Vertical asymptote: $x = 1$

Intercept: $(0, -2)$

Slant asymptote: $y = x + 1$

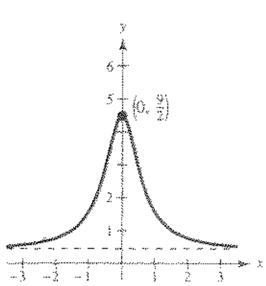


21. $f(x) = \dfrac{2x^2 + 9}{5x^2 + 2}$

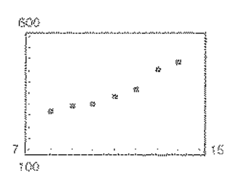
Horizontal asymptote: $y = \dfrac{2}{5}$

y-axis symmetry

Intercept: $\left(0, \dfrac{9}{2}\right)$

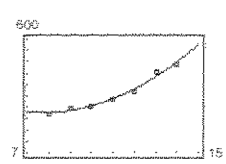


22. (a)



(b) $y = 5.582t^2 - 85.53t + 602.0$

(c)



Yes, the model is a good fit.

(d) For 2005, $t = 15$ and $y \approx \$575$ billion.

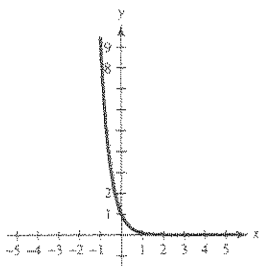
For 2010, $t = 20$ and $y \approx \$1124$ billion.

(e) Answers will vary.

Chapter 3 Chapter Test Solutions

1.

x	-2	-1	0	1	2
$f(x)$	100	10	1	0.1	0.01



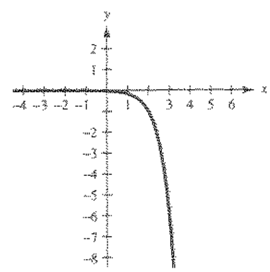
$f(x) = 10^{-x}$

Horizontal asymptote: $y = 0$

Intercept: $(0, 1)$

2.

x	0	2	3	4
$f(x)$	-0.03	-1	-6	-36



$f(x) = -6^{x-2}$

Horizontal asymptote: $y = 0$

Intercept: $\left(0, -\frac{1}{36}\right)$

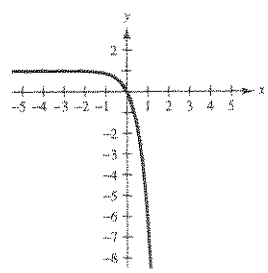
3.

x	-2	-1	0	1	2
$f(x)$	0.9817	0.8647	0	-6.3891	-53.5982

$f(x) = 1 - e^{2x}$

Horizontal asymptote: $y = 1$

Intercept: $(0, 0)$



4. $\log_7 7^{-0.89} = -0.89 \log_7 7$
$= -0.89$

5. $4.6 \ln e^2 = 4.6(2) \ln e = 9.2$

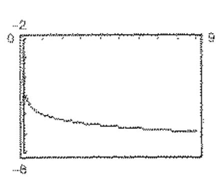
6. $2 - \log_{10} 100 = 2 - 2 = 0$

7. $f(x) = -\log_{10} x - 6$

Domain: $x > 0$

Vertical asymptote: $x = 0$

x-intercept: $(10^{-6}, 0) \approx (0, 0)$

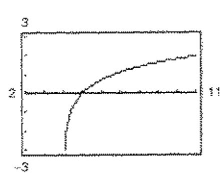


8. $f(x) = \ln(x - 4)$

Domain: $x > 4$

Vertical asymptote: $x = 4$

x-intercept: $(5, 0)$

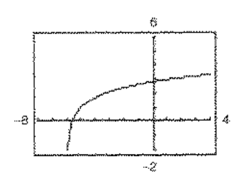


9. $f(x) = 1 + \ln(x + 6)$

Domain: $x > -6$

Vertical asymptote: $x = -6$

x-intercept: $(-5.632, 0)$



10. $\log_7 44 = \dfrac{\ln 44}{\ln 7} \approx 1.945$

11. $\log_{2/5}(0.9) = \dfrac{\ln(0.9)}{\ln(2/5)} \approx 0.115$

12. $\log_{24} 68 = \dfrac{\ln 68}{\ln 24} \approx 1.328$

13. $\log_2 3a^4 = \log_2 3 + \log_2 a^4 = \log_2 3 + 4 \log_2 a$

14. $\ln \frac{5\sqrt{x}}{6} = \ln 5 + \ln\sqrt{x} - \ln 6$

$= \ln 5 + \frac{1}{2}\ln x - \ln 6$

15. $\ln \frac{x\sqrt{x+1}}{2e^4} = \ln x + \ln \sqrt{x+1} - \ln(2) - \ln e^4$

$= \ln x + \frac{1}{2}\ln(x+1) - \ln 2 - 4$

16. $\log_3 13 + \log_3 y = \log_3(13y)$

17. $4 \ln x - 4 \ln y = \ln x^4 - \ln y^4 = \ln\left(\frac{x^4}{y^4}\right) = \ln\left(\frac{x}{y}\right)^4$

18. $\ln x - \ln(x+2) + \ln(2x-3) = \ln\left[\frac{x(2x-3)}{x+2}\right]$

19. $3^x = 81 = 3^4$

$x = 4$

20. $5^{2x} = 2500$

$2x \ln 5 = \ln 2500$

$x = \frac{1}{2}\frac{\ln 2500}{\ln 5} \approx 2.431$

21. $\log_7 x = 3$

$7^3 = x$

$x = 343$

22. $\log_{10}(x-4) = 5$

$10^5 = x - 4$

$x = 10^5 + 4$

$= 100{,}004$

23. $\frac{1025}{8 + e^{4x}} = 5$

$1025 = 40 + 5e^{4x}$

$985 = 5e^{4x}$

$e^{4x} = 197$

$4x = \ln(197)$

$x = \frac{1}{4}\ln(197) \approx 1.321$

24. $-xe^{-x} + e^{-x} = 0$

$e^{-x}(1-x) = 0$

$x = 1$

25. $\log_{10} x - \log_{10}(8-5x) = 2$

$\log_{10}\left(\frac{x}{8-5x}\right) = 2$

$10^2 = \frac{x}{8-5x}$

$800 - 500x = x$

$800 = 501x$

$x = \frac{800}{501} \approx 1.597$

26. $2x \ln x - x = 0$

$2 \ln x = 1, \quad (x \neq 0)$

$\ln x = \frac{1}{2}$

$x = e^{1/2} \approx 1.649$

27. $\frac{1}{2} = 1e^{k(22)}$ (half-life is 22 years)

$\ln \frac{1}{2} = 22k$

$k = \frac{1}{22}\ln\frac{1}{2} = -\frac{1}{22}\ln 2 \approx -0.03151$

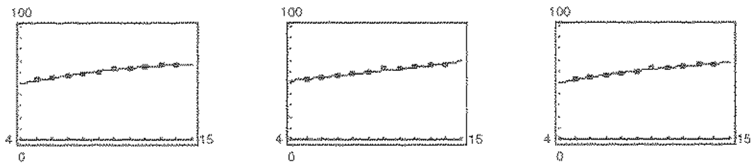
$A = e^{-0.03151(19)} \approx 0.54953$ or 55% remains

28. (a) Quadratic model: $R = -0.092t^2 + 3.29t + 38.1$

Exponential model: $R = 46.99(1.026)^t$

Power model: $R = 36.00t^{0.233}$

(b)



(c) The quadratic model is best for 2010, $t = 20$, and $R \approx 67.1$ billion dollars.

Chapters 1–3 Cumulative Test Solutions

1. (a) $$\text{Slope} = \frac{8-4}{-5-(-1)} = \frac{4}{-4} = -1$$

$$y - 8 = -1(x - (-5)) = -x - 5$$

$$x + y - 3 = 0$$

(b) Sample answers: $(0, 3), (1, 2), (2, 1)$

2. (a) $$y - 1 = -2\left(x + \frac{1}{2}\right)$$

$$y - 1 = -2x - 1$$

$$y = -2x$$

$$y + 2x = 0$$

(b) Three additional points: $(0, 0), (1, -2), (2, -4)$

3. (a) Vertical line: $x = -\frac{3}{7}$ or $x + \frac{3}{7} = 0$

(b) Three additional points:

$$\left(-\frac{3}{7}, 0\right), \left(-\frac{3}{7}, 1\right), \left(-\frac{3}{7}, 2\right)$$

4. $f(x) = \dfrac{x}{x-2}$

(a) $f(5) = \dfrac{5}{5-2} = \dfrac{5}{3}$

(b) $f(2)$ is undefined (division by 0).

(c) $f(5 + 4s) = \dfrac{5+4s}{(5+4s)-2} = \dfrac{5+4s}{3+4s}$

5. $f(x) = \begin{cases} 3x - 8, & x < 0 \\ x^2 + 4, & x \ge 0 \end{cases}$

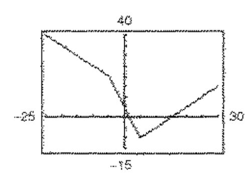
(a) $f(-8) = 3(-8) - 8 = -32$

(b) $f(0) = 0^2 + 4 = 4$

(c) $f(4) = 4^2 + 4 = 20$

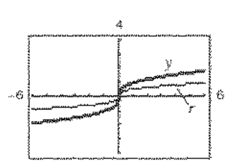
6. No, for some x there corresponds two values of y.

7.

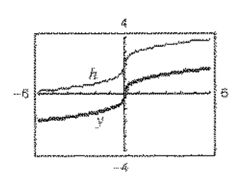


Decreasing on $(-\infty, 5)$,
Increasing on $(5, \infty)$

8. (a) $r(x) = \frac{1}{2}\sqrt[3]{x}$ is a vertical shrink of $y = \sqrt[3]{x}$.



(b) $h(x) = \sqrt[3]{x} + 2$ is a vertical shift two units upward.



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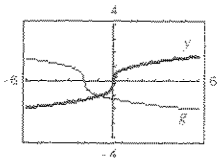
8. —CONTINUED—

(c) $f(x) = \sqrt[3]{x}$

$g(x) = -\sqrt[3]{x+2}$



g is a horizontal shift two units to the left followed by a reflection in the x-axis.

9. $(f+g)(-4) = f(-4) + g(-4) = [-(-4)^2 + 3(-4) - 10] + [4(-4) + 1]$

$= -38 - 15 = -53$

10. $(g - f)(\frac{3}{4}) = \left[4(\frac{3}{4}) + 1\right] - \left[-(\frac{3}{4})^2 + 3(\frac{3}{4}) - 10\right]$

$= 4 - (-8.3125) = 12.3125 = \frac{197}{16}$

11. $(g \circ f)(-2) = g(f(-2)) = g(-20) = 4(-20) + 1 = -79$

12. $(fg)(-1) = f(-1)g(-1) = (-14)(-3) = 42$

13. Yes, $h(x) = 5x - 2$ has an inverse function.

$y = 5x - 2$

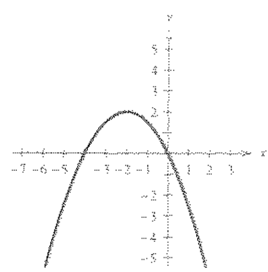
$x = 5y - 2$

$x + 2 = 5y$

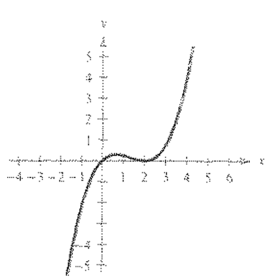
$\frac{x+2}{5} = y$

$h^{-1}(x) = \frac{x+2}{5}$

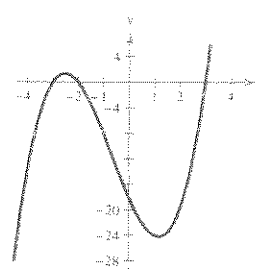
14. $f(x) = -\frac{1}{2}(x^2 + 4x)$



15. $f(x) = \frac{1}{4}x(x-2)^2$



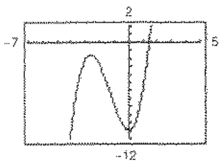
16.



17. $x^3 + 2x^2 + 4x + 8 = (x + 2)(x^2 + 4)$

Zeros: $-2, \pm 2i$

18. Using a graphing utility, $x \approx 1.424$.



19.

$$\begin{array}{r}
4x + 2 \\
x + 3 \overline{\smash{)}\,4x^2 + 14x - 9} \\
\underline{4x^2 + 12x} \\
2x - 9 \\
\underline{2x + 6} \\
-15
\end{array}$$

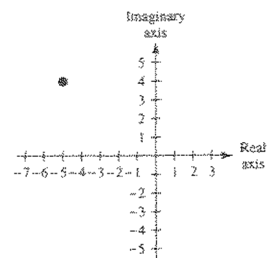
$$\frac{4x^2 + 14x - 9}{x + 3} = 4x + 2 - \frac{15}{x + 3}$$

20.

$$\begin{array}{r|rrrr}
6 & 2 & -5 & 6 & -20 \\
 & & 12 & 42 & 288 \\
\hline
 & 2 & 7 & 48 & 268
\end{array}$$

$$\frac{2x^3 - 5x^2 + 6x - 20}{x - 6} = 2x^2 + 7x + 48 + \frac{268}{x - 6}$$

21.



22. $f(x) = (x - 0)(x + 3)\left[x - \left(1 + \sqrt{5}i\right)\right]\left[x - \left(1 - \sqrt{5}i\right)\right]$

$= x(x + 3)[(x - 1)^2 + 5]$

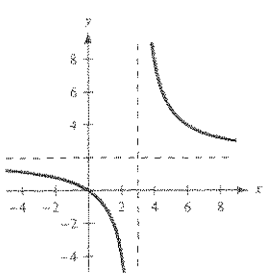
$= (x^2 + 3x)(x^2 - 2x + 6)$

$= x^4 + x^3 + 18x$

23. $f(x) = \dfrac{2x}{x - 3}$

Vertical asymptote: $x = 3$

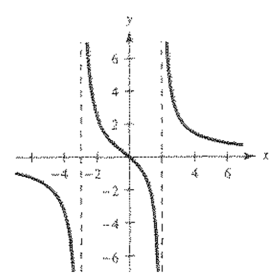
Horizontal asymptote: $y = 2$



24. $f(x) = \dfrac{5x}{x^2 + x - 6} = \dfrac{5x}{(x + 3)(x - 2)}$

Vertical asymptotes: $x = -3, 2$

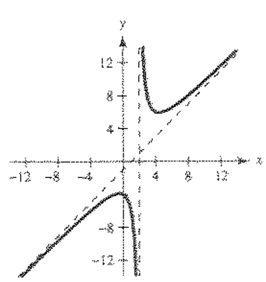
Horizontal asymptote: $y = 0$



25. $f(x) = \dfrac{x^2 - 3x + 8}{x - 2} = x - 1 + \dfrac{6}{x - 2}$

Vertical asymptote: $x = 2$

Slant asymptote: $y = x - 1$



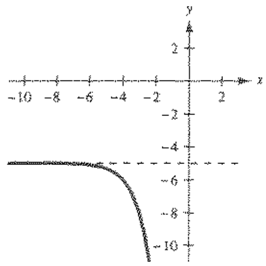
26. $(1.85)^{3.1} \approx 6.733$

27. $58^{\sqrt{5}} \approx 8772.934$

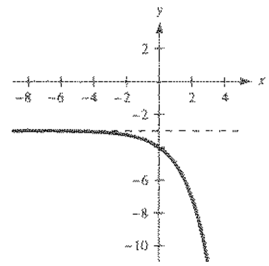
28. $e^{-20/11} \approx 0.162$

29. $4e^{2.56} \approx 51.743$

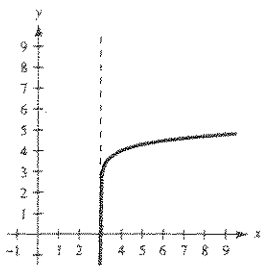
30. $f(x) = -3^{x+4} - 5$



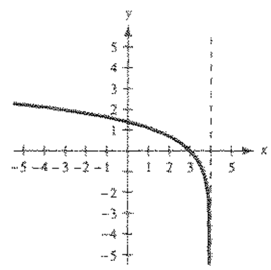
31. $f(x) = -\left(\frac{1}{2}\right)^{-x} - 3$



32. $f(x) = 4 + \log_{10}(x - 3)$



33. $f(x) = \ln(4 - x)$



34. $\log_5 21 = \dfrac{\ln 21}{\ln 5} \approx 1.892$

35. $\log_9 6.8 = \dfrac{\ln 6.8}{\ln 9} \approx 0.872$

36. $\log_2\left(\dfrac{3}{2}\right) = \dfrac{\ln\left(\frac{3}{2}\right)}{\ln 2} \approx 0.585$

37. $$\begin{aligned} \ln\left(\frac{x^2 - 4}{x^2 + 1}\right) &= \ln[(x - 2)(x + 2)] - \ln(x^2 + 1) \\ &= \ln(x - 2) + \ln(x + 2) - \ln(x^2 + 1) \end{aligned}$$

38. $2 \ln x - \ln(x - 1) + \ln(x + 1) = \ln\left[x^2\left(\dfrac{x + 1}{x - 1}\right)\right]$

39. $$\begin{aligned} 6e^{2x} &= 72 \\ e^{2x} &= 12 \\ 2x &= \ln 12 \\ x &= \frac{1}{2} \ln 12 \approx 1.242 \end{aligned}$$

40. $$\begin{aligned} 4^{x-5} + 21 &= 30 \\ 4^{x-5} &= 9 \\ (x - 5) \ln 4 &= \ln 9 \\ x - 5 &= \frac{\ln 9}{\ln 4} \\ x &= 5 + \frac{\ln 9}{\ln 4} \approx 6.585 \end{aligned}$$

41. $$\begin{aligned} \log_2 x + \log_2 5 &= 6 \\ \log_2 5x &= 6 \\ 5x &= 2^6 = 64 \\ x &= \frac{64}{5} = 12.8 \end{aligned}$$

42.
$$\begin{aligned} 250e^{0.05x} &= 500{,}000 \\ e^{0.05x} &= 2000 \\ 0.05x &= \ln 2000 \\ x &= 20 \ln 2000 \\ &\approx 152.018 \end{aligned}$$

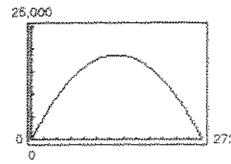
43.
$$\begin{aligned} 2x^2e^{2x} - 2xe^{2x} &= 0 \\ (2x^2 - 2x)e^{2x} &= 0 \\ 2x^2 - 2x &= 0 \\ 2x(x-1) &= 0 \\ x &= 0, 1 \end{aligned}$$

44.
$$\begin{aligned} \ln(2x-5) - \ln x &= 1 \\ \ln \frac{2x-5}{x} &= 1 \\ e &= \frac{2x-5}{x} \\ ex &= 2x - 5 \\ x(e-2) &= -5 \\ x &= \frac{-5}{e-2} < 0 \end{aligned}$$

No solution because $\ln\left(\frac{-5}{e-2}\right)$ does not exist.

45. (a) Let x and y be the lengths of the sides $2x + 2y = 546 \implies y = 273 - x$.

$A = xy = x(273 - x)$

(b) 

Domain: $0 < x < 273$

(c) If $A = 15{,}000$, then $x = 76.23$ or 196.77.

Dimensions in feet:

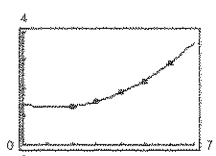
76.23×196.77 or 196.77×76.23

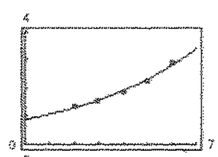
46. (a) Quadratic model: $y = 0.0707x^2 - 0.183x + 1.45$, Coefficient of determination: 0.99871

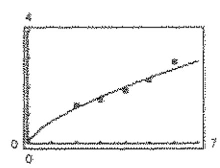
Exponential model: $y = 0.8915(1.2106)^x$, Coefficient of determination: 0.99862

Power model: $y = 0.7865x^{0.6762}$, Coefficient of determination: 0.93130

(b) Quadratic model: Exponential model: Power model:



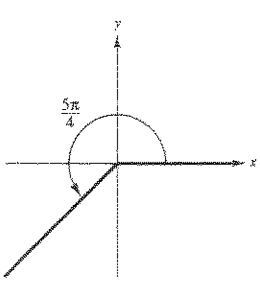




(c) The quadratic model is best because its coefficient of determination is closest to 1.

(d) For 2008, $x = 8$ and $y \approx \$4.51$. For 2010, $x = 10$ and $y \approx \$6.69$. Answers will vary.

Chapter 4 Chapter Test Solutions

1. (a) 

(b) $\frac{5\pi}{4} + 2\pi = \frac{13\pi}{4}; \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$

(c) $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 225°$

2. $90 \text{ km/hr} = \dfrac{90 \text{ km/hr}}{60 \text{ min/h}} \cdot 1000 \text{ m/km}$

$= 1500 \text{ m/min}$

$1500 = \dfrac{s}{t} = \dfrac{r\theta}{t} \Longrightarrow \text{angular speed} = \dfrac{\theta}{t} = \dfrac{1500}{r} = \dfrac{1500}{1.25/2} = 2400 \text{ rad/min}$

3. $\sin\theta = \dfrac{4}{\sqrt{17}} = \dfrac{4\sqrt{17}}{17}$ $\qquad \csc\theta = \dfrac{\sqrt{17}}{4}$

$\cos\theta = -\dfrac{1}{\sqrt{17}} = -\dfrac{\sqrt{17}}{17}$ $\qquad \sec\theta = -\sqrt{17}$

$\tan\theta = -4$ $\qquad \cot\theta = -\dfrac{1}{4}$

4. $\tan\theta = \dfrac{7}{2}$

$\tan^2\theta + 1 = \sec^2\theta \Longrightarrow \sec\theta = \sqrt{\dfrac{49}{4} + 1} = \dfrac{\sqrt{53}}{2}$

$\cos\theta = \dfrac{2}{\sqrt{53}} = \dfrac{2\sqrt{53}}{53}$

$\sin\theta = \tan\theta\cos\theta = \dfrac{7\sqrt{53}}{53}$

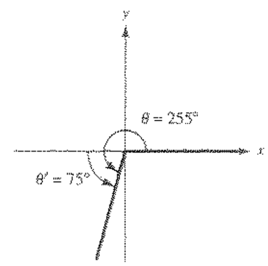
$\csc\theta = \dfrac{53}{7\sqrt{53}} = \dfrac{\sqrt{53}}{7}$

$\cot\theta = \dfrac{2}{7}$

$\sec\theta = \dfrac{\sqrt{53}}{2}$

5. $\theta = 255°$

$\theta' = 255° - 180° = 75°$



6. $\sec\theta = \dfrac{1}{\cos\theta} < 0 \Longrightarrow$ Quadrants II or III

$\tan\theta > 0 \Longrightarrow$ Quadrants I or III

Hence, Quadrant III.

7. $\cos\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = 135°, 225°$

8. $\csc\theta = \dfrac{1}{\sin\theta} = 1.030 \Longrightarrow \sin\theta = \dfrac{1}{1.030}$ and θ in Quadrant I or II.

Using a calculator, $\theta = 1.33, 1.81$ radians.

9. $\cos\theta = -\dfrac{3}{5}$, $\sin\theta > 0$, Quadrant II

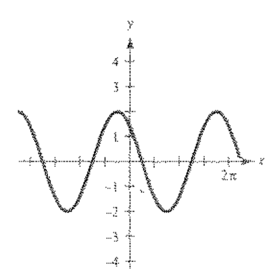
$\sin\theta = \dfrac{4}{5}$ $\qquad \tan\theta = -\dfrac{4}{3}$

$\sec\theta = -\dfrac{5}{3}$ $\qquad \csc\theta = \dfrac{5}{4}$

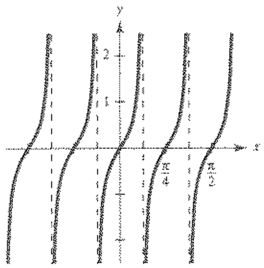
$\cot\theta = -\dfrac{3}{4}$

10. $g(x) = -2\sin\left(x - \dfrac{\pi}{4}\right)$

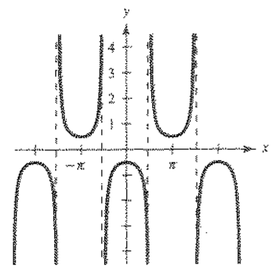
Amplitude: 2, shifted $\pi/4$ to the right



11. $f(x) = \frac{1}{2}\tan 4x$



12. $f(x) = \frac{1}{2}\sec(x - \pi)$ is $y = \frac{1}{2}\sec x$ shifted π to the right.

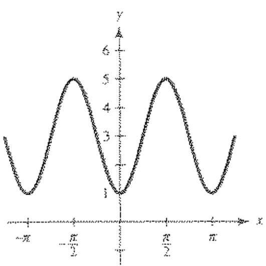


13. $f(x) = 2\cos(\pi - 2x) + 3 = 2\cos(2x - \pi) + 3$

Amplitude: 2

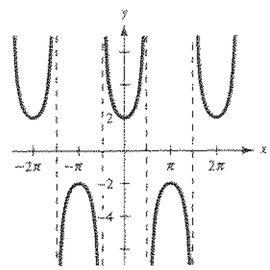
Shifted $\frac{\pi}{2}$ to the right, period π

Shifted vertically upward 3

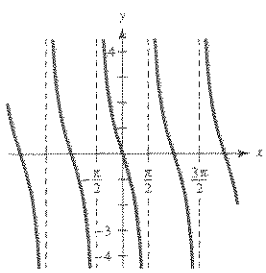


14. $f(x) = 2\csc\left(x + \frac{\pi}{2}\right)$

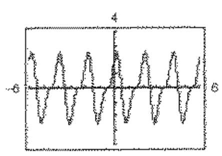
Shifted $\frac{\pi}{2}$ to the left



15. $f(x) = 2\cot\left(x - \frac{\pi}{2}\right)$

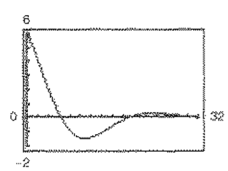


16.



Period is 2.

17.



Not periodic

18. Amplitude: 2

Reflected in x-axis $\Longrightarrow a = -2$

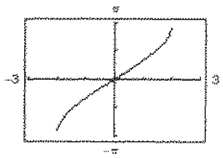
Period 4π and shifted to the right:

$y = -2\sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$

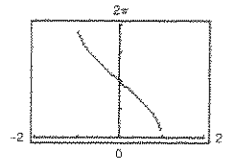
19. Let $u = \arccos\frac{2}{3} \Longrightarrow \cos u = \frac{2}{3}$.

Then $\tan\left(\arccos\frac{2}{3}\right) = \tan u = \frac{\sqrt{5}}{2}$.

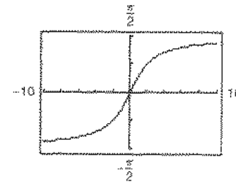
20. $f(x) = 2 \arcsin\left(\frac{1}{2}x\right)$



21. $f(x) = 2 \arccos x$

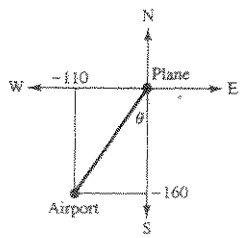


22. $f(x) = \arctan\left(\frac{x}{2}\right)$



23. $\tan \theta = \frac{110}{160} \implies \theta \approx 34.5^\circ$

Bearing: 214.5°



Chapter 5 Chapter Test Solutions

1. $\tan \theta = \frac{3}{2}, \cos \theta < 0 \implies \theta$ in Quadrant III

$$\sec^2 \theta = \tan^2 \theta + 1 = \frac{9}{4} + 1 = \frac{13}{4} \implies \sec \theta = -\frac{\sqrt{13}}{2}$$

$$\cos \theta = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\sin \theta = \tan \theta \cos \theta = \frac{3}{2}\left(-\frac{2\sqrt{13}}{13}\right) = -\frac{3\sqrt{13}}{13}$$

$$\csc \theta = -\frac{13}{3\sqrt{13}} = -\frac{\sqrt{13}}{3}$$

$$\cot \theta = \frac{2}{3}$$

2. $\csc^2 \beta(1 - \cos^2 \beta) = \dfrac{1}{\sin^2 \beta} \cdot \sin^2 \beta = 1$

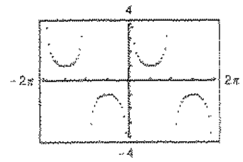
3. $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} = \dfrac{[(\sec^2 x) + (\tan^2 x)][\sec^2 x - \tan^2 x]}{\sec^2 x + \tan^2 x} = \sec^2 x - \tan^2 x = 1$

4. $\dfrac{\cos \theta}{\sin \theta} + \dfrac{\sin \theta}{\cos \theta} = \dfrac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \dfrac{1}{\sin \theta \cos \theta} = \csc \theta \sec \theta$

5. Since $\tan^2 \theta = \sec^2 \theta - 1$ for all θ, then $\tan \theta = -\sqrt{\sec^2 \theta - 1}$ in Quadrants II and IV.

Thus, $\frac{\pi}{2} < \theta \le \pi$ and $\frac{3\pi}{2} < \theta < 2\pi$.

6. Conjecture: $y_1 = y_2$



Algebraically,

$$y_1 = \sin x + \cos x \cot x = \sin x + \frac{\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x}$$

$$= \frac{1}{\sin x} = \csc x.$$

7. $\sin\theta \cdot \sec\theta = \sin\theta \dfrac{1}{\cos\theta} = \tan\theta$

8. $\sec^2 x \tan^2 x + \sec^2 x = \sec^2 x(\tan^2 x + 1) = \sec^4 x$

9. $$\frac{\csc\alpha + \sec\alpha}{\sin\alpha + \cos\alpha} = \frac{\dfrac{1}{\sin\alpha} + \dfrac{1}{\cos\alpha}}{\sin\alpha + \cos\alpha} = \frac{\dfrac{\cos\alpha + \sin\alpha}{\sin\alpha \cdot \cos\alpha}}{(\sin\alpha + \cos\alpha)}$$

$$= \frac{1}{\sin\alpha\cos\alpha} = \frac{\cos^2\alpha + \sin^2\alpha}{\sin\alpha\cos\alpha}$$

$$= \frac{\cos\alpha}{\sin\alpha} + \frac{\sin\alpha}{\cos\alpha} = \cot\alpha + \tan\alpha$$

10. $$\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$$

$$= 0 - \sin x = -\sin x$$

11. $$\sin(n\pi + \theta) = \sin n\pi\cos\theta + \cos n\pi \sin\theta$$

$$= 0 + (-1)^n \sin\theta$$

$$= (-1)^n \sin\theta$$

12. $$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$$

$$= 1 + \sin 2x$$

13. $$\tan 105° = \tan(45° + 60°)$$

$$= \frac{\tan 45° + \tan 60°}{1 - \tan 45° \tan 60°}$$

$$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} = -2 - \sqrt{3}$$

14. $$\sin^4 x \tan^2 x = \frac{\sin^6 x}{\cos^2 x}$$

$$= \frac{1}{32} \cdot \frac{(10 - 15\cos 2x + 6\cos 4x - \cos 6x)}{(1 + \cos 2x)/2}$$

$$= \frac{1}{16}\left[\frac{10 - 15\cos 2x + 6\cos 4x - \cos 6x}{1 + \cos 2x}\right]$$

15. $$\frac{\sin 4\theta}{1 + \cos 4\theta} = \tan\frac{4\theta}{2} = \tan 2\theta$$

16. $$4\cos 2\theta \sin 4\theta = 4\left(\frac{1}{2}\right)[\sin(2\theta + 4\theta) - \sin(2\theta - 4\theta)]$$

$$= 2[\sin 6\theta - \sin(-2\theta)]$$

$$= 2[\sin 6\theta + \sin 2\theta]$$

17. $\sin 3\theta - \sin 4\theta = 2\cos\left(\frac{3\theta + 4\theta}{2}\right)\sin\left(\frac{3\theta - 4\theta}{2}\right)$

$$= 2\cos\left(\frac{7\theta}{2}\right)\sin\left(\frac{-\theta}{2}\right)$$

$$= -2\cos\frac{7\theta}{2}\sin\frac{\theta}{2}$$

18. $\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0 \implies x = 0, \pi$

$\tan x + 1 = 0 \implies \tan x = -1 \implies x = \frac{3\pi}{4}, \frac{7\pi}{4}$

19. $\sin 2\alpha - \cos\alpha = 0$

$2\sin\alpha\cos\alpha - \cos\alpha = 0$

$\cos\alpha(2\sin\alpha - 1) = 0$

$\cos\alpha = 0 \implies a = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\sin\alpha - 1 = 0 \implies \sin a = \frac{1}{2} \implies \alpha = \frac{\pi}{6}, \frac{5\pi}{6}$

20. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

21. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0 \implies \csc x = 2 \implies \sin x = \frac{1}{2} \implies x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\csc x + 1 = 0 \implies \csc x = -1 \implies \sin x = -1 \implies x = \frac{3\pi}{2}$

22. $3\cos x - x = 0$

$x \approx 1.170, -2.663, -2.938$

23. $\sin 2u = 2\sin u\cos u = 2\frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}} = \frac{4}{5}$

$\tan 2u = \frac{2\tan u}{1 - \tan^2 u} = \frac{2(2)}{1 - 2^2} = -\frac{4}{3}$

$\cos 2u = 1 - 2\sin^2 u = 1 - 2\left(\frac{2}{\sqrt{5}}\right)^2 = -\frac{3}{5}$

24. $n = \frac{\sin[(\theta/2) + (\alpha/2)]}{\sin(\theta/2)}$

$\frac{3}{2} = \frac{\sin[(\theta/2) + 30°]}{\sin(\theta/2)}$

$3\sin\frac{\theta}{2} = 2\left[\sin\frac{\theta}{2}\cos 30° + \cos\frac{\theta}{2}\sin 30°\right]$

$3\sin\frac{\theta}{2} = \sqrt{3}\sin\frac{\theta}{2} + \cos\frac{\theta}{2}$

$\left(3 - \sqrt{3}\right)\sin\frac{\theta}{2} = \cos\frac{\theta}{2}$

$\tan\frac{\theta}{2} = \frac{1}{3 - \sqrt{3}}$

$\frac{\theta}{2} = \arctan\left(\frac{1}{3 - \sqrt{3}}\right) \approx 38.26°$

$\theta \approx 76.52°$

Chapter 6 Chapter Test Solutions

1. $A = 36°, B = 98°, c = 16$

$C = 180° - 36° - 98° = 46°$

$a = \frac{c}{\sin C} \sin A \approx 13.07$

$b = \frac{a}{\sin A} \sin B \approx 22.03$

2. $a = 4, b = 8, c = 10$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{-20}{64} \implies C \approx 108.21°$

$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{52}{80} \implies B \approx 49.46°$

$A = 180° - C - B \approx 22.33°$

3. $A = 35°, b = 8, c = 12$

$a^2 = b^2 + c^2 - 2\,bc \cdot \cos A$

$= 64 + 144 - 2(8)(12) \cos 35° \approx 50.7228$

$a \approx 7.12$

$\sin B = \frac{\sin A}{a} b \approx 0.6443 \implies B \approx 40.11°$

$C = 180° - A - B \approx 104.89°$

4. $A = 25°, b = 28, a = 18$

$\sin B = \frac{\sin A}{a} b = \frac{(\sin 25°)28}{18} \approx 0.6574$

$B_1 \approx 41.10°$ or $B_2 \approx 180° - 41.1° \approx 138.90°$

Case 1: $B_1 = 41.10°$

$C = 180° - A - B_1 \approx 113.90°$

$c = \frac{a}{\sin A} \sin C \approx 38.94$

Case 2: $B_2 = 138.90°$

$C = 180° - A - B_2 \approx 16.10°$

$c = \frac{a}{\sin A} \sin C \approx 11.81$

5. No triangle possible $(5.2 \le 10.1)$

6. $\sin B = \frac{\sin A}{a} b = \frac{\sin 150°}{9.4} 4.8$

$\approx 0.2553 \implies B \approx 14.8°$

$C = 180° - A - B = 15.2°$

$c = \frac{a}{\sin A} \sin C \approx 4.9$

7. Law of Cosines:

$a^2 = b^2 + c^2 - 2bc \cos \theta$

$= 480^2 + 565^2 - 2(480)(565) \cos 80°$

$= 455{,}438.2 \implies a \approx 674.9$ ft

8. $s = \frac{a + b + c}{2} = \frac{55 + 85 + 100}{2} = 120$

$A = \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{120(65)(35)(20)}$

≈ 2336.7 square meters

9. $\mathbf{w} = \langle 4 - (-8), 1 - (-12) \rangle = \langle 12, 13 \rangle$

$\|\mathbf{w}\| = \sqrt{12^2 + 13^2} = \sqrt{313} \approx 17.69$

10. (a) $2\mathbf{v} + \mathbf{u} = 2\langle -2, -8 \rangle + \langle 0, -4 \rangle = \langle -4, -20 \rangle$

(b) $\mathbf{u} - 3\mathbf{v} = \langle 0, -4 \rangle - 3\langle -2, -8 \rangle = \langle 6, 20 \rangle$

(c) $5\mathbf{u} - \mathbf{v} = 5\langle 0, -4 \rangle - \langle -2, -8 \rangle = \langle 2, -12 \rangle$

11. (a) $2\mathbf{v} + \mathbf{u} = 2\langle -1, -10 \rangle + \langle -2, -3 \rangle$

$= \langle -4, -23 \rangle$

(b) $\mathbf{u} - 3\mathbf{v} = \langle -2, -3 \rangle - 3\langle -1, -10 \rangle = \langle 1, 27 \rangle$

(c) $5\mathbf{u} - \mathbf{v} = 5\langle -2, -3 \rangle - \langle -1, -10 \rangle$

$= \langle -9, -5 \rangle$

12. (a) $2\mathbf{v} + \mathbf{u} = 2(6\mathbf{i} + 9\mathbf{j}) + (\mathbf{i} - \mathbf{j}) = 13\mathbf{i} + 17\mathbf{j}$

(b) $\mathbf{u} - 3\mathbf{v} = (\mathbf{i} - \mathbf{j}) - 3(6\mathbf{i} + 9\mathbf{j}) = -17\mathbf{i} - 28\mathbf{j}$

(c) $5\mathbf{u} - \mathbf{v} = 5(\mathbf{i} - \mathbf{j}) - (6\mathbf{i} + 9\mathbf{j}) = -\mathbf{i} - 14\mathbf{j}$

13. (a) $2\mathbf{v} + \mathbf{u} = 2(-\mathbf{i} - 2\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = -\mathbf{j}$

(b) $\mathbf{u} - 3\mathbf{v} = (2\mathbf{i} + 3\mathbf{j}) - 3(-\mathbf{i} - 2\mathbf{j}) = 5\mathbf{i} + 9\mathbf{j}$

(c) $5\mathbf{u} - \mathbf{v} = 5(2\mathbf{i} + 3\mathbf{j}) - (-\mathbf{i} - 2\mathbf{j}) = 11\mathbf{i} + 17\mathbf{j}$

14. Unit vector $= \dfrac{\mathbf{v}}{\|\mathbf{v}\|} = \dfrac{1}{\sqrt{49+16}}\langle 7, 4\rangle$

$= \left\langle \dfrac{7}{\sqrt{65}}, \dfrac{4}{\sqrt{65}} \right\rangle = \dfrac{\sqrt{65}}{65}\langle 7, 4\rangle$

15. $12\dfrac{\langle 3, -5\rangle}{\|\langle 3, -5\rangle\|} = \dfrac{12}{\sqrt{34}}\langle 3, -5\rangle = \left\langle \dfrac{36}{\sqrt{34}}, \dfrac{-60}{\sqrt{34}} \right\rangle$

$= \left\langle \dfrac{18\sqrt{34}}{17}, \dfrac{-30\sqrt{34}}{17} \right\rangle$

16. $250(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$, first force

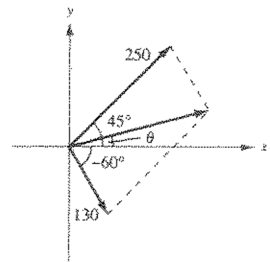
$130(\cos(-60°)\mathbf{i} + \sin(-60°)\mathbf{j})$, second force

Resultant: $\left[250\left(\dfrac{\sqrt{2}}{2}\right) + 130\left(\dfrac{1}{2}\right)\right]\mathbf{i} + \left[250\left(\dfrac{\sqrt{2}}{2}\right) + 130\left(-\dfrac{\sqrt{3}}{2}\right)\right]\mathbf{j}$

$= \left(125\sqrt{2} + 65\right)\mathbf{i} + \left(125\sqrt{2} - 65\sqrt{3}\right)\mathbf{j}$

Magnitude: $\sqrt{\left(125\sqrt{2} + 65\right)^2 + \left(125\sqrt{2} - 65\sqrt{3}\right)^2} \approx 250.15$

Direction: $\theta = \arctan\left(\dfrac{125\sqrt{2} - 65\sqrt{3}}{125\sqrt{2} + 65}\right) \implies \theta \approx 14.9°$



17. $\mathbf{u} \cdot \mathbf{v} = \langle -9, 4\rangle \cdot \langle 1, 3\rangle = -9 + 12 = 3$

18. $\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{-8}{\sqrt{53}(4)} \implies \theta \approx 105.9°$

19. No, the dot product is 24, not 0.

20. $\text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{-37}{26}\langle -5, -1\rangle = \left\langle \dfrac{185}{26}, \dfrac{37}{26} \right\rangle = \mathbf{w}_1$

$\mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 = \langle 6, 7\rangle - \left\langle \dfrac{185}{26}, \dfrac{37}{26} \right\rangle = \left\langle \dfrac{-29}{26}, \dfrac{145}{26} \right\rangle$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$

21. $|z| = 2\sqrt{2}$

$z = 2\sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

22. $100(\cos 240° + i\sin 240°) = -50 - 50\sqrt{3}i$

23. $\left[3\left(\cos\dfrac{5\pi}{6} + i\sin\dfrac{5\pi}{6}\right)\right]^8 = 3^8\left(\cos\dfrac{40\pi}{6} + i\sin\dfrac{40\pi}{6}\right)$

$= 3^8\left(-\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)$

$= -3280.5 + 3280.5\sqrt{3}i = -\dfrac{6561}{2} + \dfrac{6561}{2}\sqrt{3}i$

24. $(3 - 3i)^6 = \left[3\sqrt{2}\left(\cos\dfrac{7\pi}{4} + i\sin\dfrac{7\pi}{4}\right)\right]^6$

$= 5832\left(\cos\dfrac{42\pi}{4} + i\sin\dfrac{42\pi}{4}\right) = 5832i$

25. $128(1 + \sqrt{3}i) = 256\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 256\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$

Fourth roots: $\sqrt[4]{256}\left(\cos\frac{(\pi/3) + 2\pi k}{4} + i \sin\frac{(\pi/3) + 2k\pi}{4}\right),\ k = 0, 1, 2, 3$

Four roots are: $4\left(\cos\frac{\pi}{12} + i \sin\frac{\pi}{12}\right) \approx 3.8637 + 1.0353i$

$4\left(\cos\frac{7\pi}{12} + i \sin\frac{7\pi}{12}\right) \approx -1.0353 + 3.8637i$

$4\left(\cos\frac{13\pi}{12} + i \sin\frac{13\pi}{12}\right) \approx -3.8637 - 1.0353i$

$4\left(\cos\frac{19\pi}{12} + i \sin\frac{19\pi}{12}\right) \approx 1.0353 - 3.8637i$

26. $x^4 = 625i$

Fourth roots of $625i = 625\left(\cos\frac{\pi}{2} + i \sin\frac{\pi}{2}\right)$

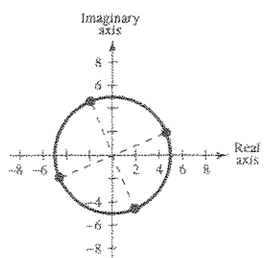
$\sqrt[4]{625}\left(\cos\left(\frac{(\pi/2) + 2\pi k}{4}\right) + i \sin\left(\frac{(\pi/2) + 2\pi k}{4}\right)\right),\ k = 0, 1, 2, 3$

Four roots are: $5\left(\cos\frac{\pi}{8} + i \sin\frac{\pi}{8}\right)$

$5\left(\cos\frac{5\pi}{8} + i \sin\frac{5\pi}{8}\right)$

$5\left(\cos\frac{9\pi}{8} + i \sin\frac{9\pi}{8}\right)$

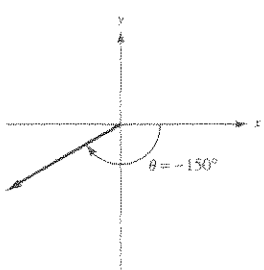
$5\left(\cos\frac{13\pi}{8} + i \sin\frac{13\pi}{8}\right)$



Chapters 4–6 Cumulative Test Solutions

1. $\theta = -150°$

(a)



(b) $-150° + 360° = 210°$

(c) $(-150°)\frac{\pi}{180°} = -\frac{5}{6}\pi$ radians

(d) $\theta' = 30°$

(e) $\sin\theta = -\frac{1}{2}$ $\quad\csc\theta = -2$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\quad\sec\theta = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$ $\quad\cot\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

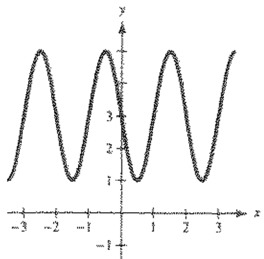
2. $2.55 \text{ rad} = 2.55\left(\frac{180°}{\pi}\right) \approx 146.1°$

3. $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(-\frac{12}{5}\right)^2 = \frac{169}{25}$

$\sec\theta = -\frac{13}{5}$ (since θ in Quadrant II)

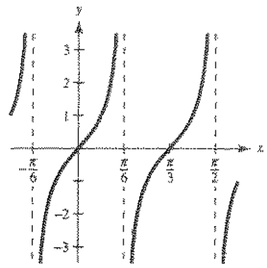
$\cos\theta = -\frac{5}{13}$

4. $f(x) = 3 - 2\sin \pi x$

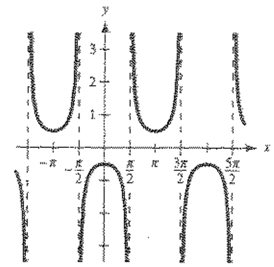


5. $f(x) = \tan(3x)$

Period: $\frac{\pi}{3}$



6. $f(x) = \frac{1}{2}\sec(x + \pi)$



7. Amplitude: 3

Cosine curve reflected about the x-axis

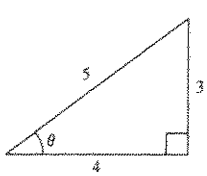
Period: $2 \implies h(x) = -3\cos(\pi x)$

Answer: $a = -3, b = \pi, c = 0$

8. Let $\theta = \arctan\frac{3}{4}$.

$\tan\theta = \frac{3}{4}$

$\sin\left(\arctan\frac{3}{4}\right) = \frac{3}{5}$



9. Let $\arcsin\left(-\frac{1}{2}\right) = \theta$.

$\sin\theta = -\frac{1}{2}$

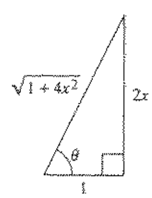
$\theta = -\frac{\pi}{6}$

$\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$

10. Let $\theta = \arctan 2x$.

$\tan\theta = 2x$

$\sin(\arctan 2x) = \sin\theta = \frac{2x}{\sqrt{1+4x^2}}$



11.
$$\frac{\sin\theta - 1}{\cos\theta} - \frac{\cos\theta}{\sin\theta - 1} = \frac{\sin^2\theta - 2\sin\theta + 1 - \cos^2\theta}{\cos\theta(\sin\theta - 1)}$$
$$= \frac{\sin^2\theta - 2\sin\theta + \sin^2\theta}{\cos\theta(\sin\theta - 1)}$$
$$= \frac{2\sin\theta(\sin\theta - 1)}{\cos\theta(\sin\theta - 1)} = 2\tan\theta$$

12. $\cot^2\alpha(\sec^2\alpha - 1) = \cot^2\alpha(\tan^2\alpha) = 1$

13. $$\begin{aligned}\sin(x+y)\sin(x-y) &= [\sin x\cos y + \cos x \sin y][\sin x \cos y - \sin y \cos x]\\ &= \sin^2 x\cos^2 y - \sin^2 y\cos^2 x\\ &= \sin^2 x(1-\sin^2 y) - \sin^2 y(1-\sin^2 x)\\ &= \sin^2 x - \sin^2 x\sin^2 y - \sin^2 y + \sin^2 y \sin^2 x\\ &= \sin^2 x - \sin^2 y\end{aligned}$$

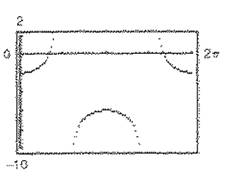
14. $$\begin{aligned}\sin^2 x\cos^2 x &= \frac{1}{4}(2\sin x\cos x)^2\\ &= \frac{1}{4}(\sin 2x)^2\\ &= \frac{1}{4}\cdot\frac{1-\cos 4x}{2}\\ &= \frac{1}{8}(1-\cos 4x)\end{aligned}$$

15. $$\begin{aligned}\sin^2 x + 2\sin x + 1 &= 0\\ (\sin x + 1)^2 &= 0\\ \sin x + 1 &= 0\\ \sin x &= -1\\ x &= \frac{3\pi}{2} + 2n\pi\end{aligned}$$

16. $$\begin{aligned}3\tan\theta - \cot\theta &= 0\\ 3\tan\theta - \frac{1}{\tan\theta} &= 0\\ 3\tan^2\theta - 1 &= 0\\ \tan\theta = \pm\frac{1}{\sqrt{3}} &\implies \theta = \frac{\pi}{6} + n\pi, \frac{5\pi}{6} + n\pi\end{aligned}$$

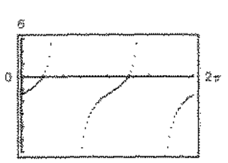
17. Graph $y = \cos^2 x - 5\cos x - 1$ on $[0, 2\pi)$.

Roots are $x \approx 1.7646, 4.5186$.

18. 

Zeros: $x \approx 1.047, 5.236$

Algebraically: $$\begin{aligned}\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} &= 4\\ \frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1+\sin x)} &= 4\\ \frac{2 + 2\sin x}{\cos x(1+\sin x)} &= 4\\ \frac{2}{\cos x} &= 4\\ \cos x &= \frac{1}{2}\\ x &= \frac{\pi}{3}, \frac{5\pi}{3}\end{aligned}$$

19. 

Zeros: $x \approx 0.785, 3.927$

Algebraically: $$\begin{aligned}\tan^3 x - \tan^2 x + 3\tan x - 3 &= 0\\ \tan^2 x(\tan x - 1) + 3(\tan x - 1) &= 0\\ (\tan^2 x + 3)(\tan x - 1) &= 0\\ \tan x = 1 \implies x &= \frac{\pi}{4}, \frac{5\pi}{4}\end{aligned}$$

20. $\sin u = \dfrac{12}{13} \implies \cos u = \dfrac{5}{13} \implies \tan u = \dfrac{12}{5}$

$\cos v = \dfrac{3}{5} \implies \sin v = \dfrac{4}{5} \implies \tan v = \dfrac{4}{3}$

$\tan(u - v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v} = \dfrac{(12/5) - (4/3)}{1 + (12/5)(4/3)} = \dfrac{16}{63}$

21. $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta} = \dfrac{2(1/2)}{1 - (1/4)} = \dfrac{4}{3}$

22. $\sec^2\theta = \tan^2\theta + 1 = \dfrac{25}{9} \implies \sec\theta = -\dfrac{5}{3} \implies \cos\theta = -\dfrac{3}{5}$ (Quadrant III)

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1 - \cos\theta}{2}} = \pm\dfrac{2}{\sqrt{5}} = \pm\dfrac{2\sqrt{5}}{5}$

$\dfrac{\theta}{2}$ in Quadrant II: $\sin\dfrac{\theta}{2} = \dfrac{2\sqrt{5}}{5}$

23. $\cos 8x + \cos 4x = 2\cos\left(\dfrac{8x + 4x}{2}\right)\cos\left(\dfrac{8x - 4x}{2}\right)$

$= 2\cos 6x \cos 2x$

24. $\tan x(1 - \sin^2 x) = \dfrac{\sin x}{\cos x}\cos^2 x = \sin x \cos x$

$= \dfrac{1}{2}(2\sin x\cos x) = \dfrac{1}{2}\sin 2x$

25. $\sin 3\theta \sin\theta = \frac{1}{2}[\cos(3\theta - \theta) - \cos(3\theta + \theta)]$

$= \frac{1}{2}(\cos 2\theta - \cos 4\theta)$

26. $\sin 3x \cos 2x = \frac{1}{2}(\sin(3x + 2x) + \sin(3x - 2x))$

$= \frac{1}{2}(\sin 5x + \sin x)$

27. $\dfrac{2\cos 3x}{\sin 4x - \sin 2x} = \dfrac{2\cos 3x}{2\cos 3x \cdot \sin x} = \dfrac{1}{\sin x} = \csc x$

28. $\sin B = \dfrac{\sin A}{a}b = 0.2569 \implies B \approx 14.9°$

$C = 180° - 46° - 14.9° = 119.1°$

$c = \dfrac{a}{\sin A}(\sin C) \approx 17.0$

29. $A = 32°, b = 8, c = 10$

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 64 + 100 - 2(8)(10)\cos 32°$

$= 28.3123 \implies a \approx 5.32$

$\sin B = \dfrac{b\sin A}{a} \approx 0.7967 \implies B \approx 52.82°$

$C = 180° - B - A = 95.18°$

30. $B = 180° - 24° - 101° = 55°$

$b = \dfrac{a}{\sin A}\sin B \approx 20.14$

$c = \dfrac{a}{\sin A}\sin C \approx 24.13$

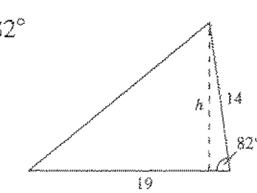
31. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = 0.8982 \implies A \approx 26.1°$

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac} = 0.8355 \implies B \approx 33.3°$

$C = 180° - 26.1° - 33.3° = 120.6°$

32. $A = \dfrac{1}{2}bh = \dfrac{1}{2}\,19 \cdot 14\sin 82°$

≈ 131.7 sq. in.



33. $s = \dfrac{12 + 16 + 18}{2} = 23$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{23(11)(7)(5)} \approx 94.10$ sq. in.

34. $\mathbf{u} = \langle 3, 5\rangle = 3\mathbf{i} + 5\mathbf{j}$

35. $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$

$\|\mathbf{v}\| = \sqrt{1+4} = \sqrt{5}$

Unit vector: $\dfrac{1}{\sqrt{5}}\mathbf{i} - \dfrac{2}{\sqrt{5}}\mathbf{j} = \left\langle \dfrac{\sqrt{5}}{5}, \dfrac{-2\sqrt{5}}{5}\right\rangle$

36. $\mathbf{u} \cdot \mathbf{v} = 3(1) + 4(-2) = -5$

37.

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= 0 \\ \langle 1, 2k\rangle \cdot \langle 2, -1\rangle &= 0 \\ 2 - 2k &= 0 \\ k &= 1 \end{aligned}$$

38. $\text{proj}_{\mathbf{v}}\mathbf{u} = \dfrac{8-10}{26}\langle 1, 5\rangle = \left\langle -\dfrac{1}{13}, -\dfrac{5}{13}\right\rangle = \mathbf{w}_1$

$$\begin{aligned} \mathbf{w}_2 = \mathbf{u} - \mathbf{w}_1 &= \langle 8, -2\rangle - \left\langle -\frac{1}{13}, -\frac{5}{13}\right\rangle \\ &= \left\langle \frac{105}{13}, -\frac{21}{13}\right\rangle \end{aligned}$$

$\mathbf{u} = \mathbf{w}_1 + \mathbf{w}_2$

39. $|z| = 3\sqrt{2}$, $\theta = \dfrac{3\pi}{4}$: $3\sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$

40. $8\left(-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i\right) = -4\sqrt{3} + 4i$

41.

$$\begin{aligned} [4(\cos 30^\circ + i\sin 30^\circ)][6(\cos 120^\circ + i\sin 120^\circ)] &= 24(\cos(30 + 120^\circ) + i\sin(30^\circ + 120^\circ)) \\ &= 24(\cos 150^\circ + i\sin 150^\circ) \\ &= 24\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -12\sqrt{3} + 12i \end{aligned}$$

42. $2 + i = \sqrt{5}(\cos\theta + i\sin\theta)$, where $\tan\theta = \dfrac{1}{2} \Longrightarrow \theta \approx 0.4636$

Square roots: $z_1 = 5^{1/4}\left(\cos\dfrac{\theta}{2} + i\sin\dfrac{\theta}{2}\right)$

$z_2 = 5^{1/4}\left(\cos\left(\dfrac{\theta + 2\pi}{2}\right) + i\sin\left(\dfrac{\theta+2\pi}{2}\right)\right)$

$z_1 \approx 1.4533 + 0.3436i$

$z_2 \approx -1.4553 - 0.3436i$

43. $1 = 1(\cos 0 + i\sin 0)$

$\cos\left(\dfrac{0 + 2\pi k}{3}\right) + i\sin\left(\dfrac{0 + 2\pi k}{3}\right)$, $k = 0, 1, 2$

$k = 0$: $\cos 0 + i\sin 0 = 1$

$k = 1$: $\cos\dfrac{2\pi}{3} + i\sin\dfrac{2\pi}{3} = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

$k = 2$: $\cos\dfrac{4\pi}{3} + i\sin\dfrac{4\pi}{3} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$

44. $x^5 = -243$

Five fifth roots of $-243 = 243(\cos \pi + i \sin \pi)$ are:

$$\sqrt[5]{243}\left(\cos \frac{\pi + 2\pi k}{5} + i \sin \frac{\pi + 2\pi k}{5}\right);\ k = 0, 1, 2, 3, 4$$

$$3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$$

$$3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$$

$$3\left(\cos \frac{5\pi}{5} + i \sin \frac{5\pi}{5}\right) = -3$$

$$3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$$

$$3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$$

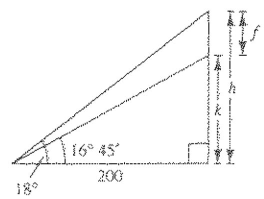
45. $\tan 18° = \dfrac{h}{200}$

$\tan 16° 45' = \dfrac{k}{200}$

Hence,

$$f = h - k = 200 \tan 18° - 200 \tan 16° 45'$$

$$\approx 4.8 \approx 5 \text{ feet.}$$



Not drawn to scale

46. $y = 4 \cos\left(\dfrac{\pi}{4}t\right)$ or $y = 4 \sin\left(\dfrac{\pi}{4}t\right)$

Amplitude: 4

Period $\dfrac{2\pi}{(\pi/4)} = 8$

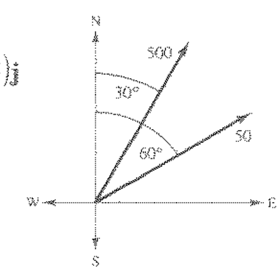
47. Add the two vectors:

$$500(\cos 60°\mathbf{i} + \sin 60°\mathbf{j}) + 50(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = \left(250 + 25\sqrt{3}\right)\mathbf{i} + \left(250\sqrt{3} + 25\right)\mathbf{j}$$

$$\tan \theta = \frac{250\sqrt{3} + 25}{250 + 25\sqrt{3}} \approx 1.56 \implies \theta \approx 57.4°$$

Direction: N 32.6° E or 32.6° in airplane navigation

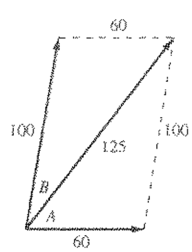
$$\text{Speed} = \sqrt{\left(250 + 25\sqrt{3}\right)^2 + \left(250\sqrt{3} + 25\right)^2} \approx 543.9 \text{ km/hr}$$



48. $\cos A = \dfrac{60^2 + 125^2 - 100^2}{2(60)(125)} = 0.615 \implies A \approx 52.05°$

$\cos B = \dfrac{100^2 + 125^2 - 60^2}{2(100)(125)} = 0.881 \implies B \approx 28.24$

Angle between vectors $= A + B \approx 80.3°$



Chapter 7 Chapter Test Solutions

1. $x - y = 6 \implies y = x - 6$. Then

$3x + 5(x - 6) = 2 \implies$

$8x = 32 \implies x = 4, y = 4 - 6 = -2.$

Answer: $(4, -2)$

2. $y = x - 1 = (x - 1)^3 \implies x = 1$ or

$1 = (x - 1)^2 = x^2 - 2x + 1 \implies x^2 - 2x = 0.$

Thus, $x = 1$ or $x(x - 2) = 0 \implies x = 0, 1, 2.$

Answer: $(0, -1), (1, 0), (2, 1)$

3. $x - y = 3 \implies y = x - 3 \implies 4x - (x-3)^2 = 7$

$$4x - (x^2 - 6x + 9) = 7$$
$$x^2 - 10x + 16 = 0$$
$$(x-2)(x-8) = 0$$

$x = 2, 8$

Answer: $(2, -1), (8, 5)$

4. $\begin{cases} 2x + 5y = -11 & \text{Equation 1} \\ 5x - y = 19 & \text{Equation 2} \end{cases}$

$-\frac{5}{2}$ times Eq. 1 added to Eq. 2 produces

$-\frac{27}{2}y = \frac{93}{2} \implies y = -\frac{31}{9}.$

Then $2x + 5\left(-\frac{31}{9}\right) = -11 \implies x = \frac{28}{9}.$

Answer: $\left(\frac{28}{9}, -\frac{31}{9}\right)$

5. $\begin{cases} 3x - 2y + z = 0 \\ 6x + 2y + 3z = -2 \\ 3x - 4y + 5z = 5 \end{cases}$

$\begin{cases} 3x - 2y + z = 0 \\ 6y + z = -2 \\ -2y + 4z = 5 \end{cases}$

$\begin{cases} 3x - 2y + z = 0 \\ y - 2z = -\frac{5}{2} \\ 13z = 13 \end{cases}$

$z = 1$

$y = 2(1) - \frac{5}{2} = -\frac{1}{2}$

$x = \frac{1}{3}\left(-1 + 2\left(-\frac{1}{2}\right)\right) = \frac{1}{3}(-2) = -\frac{2}{3}$

Answer: $\left(-\frac{2}{3}, -\frac{1}{2}, 1\right)$

6. $\begin{cases} x - 4y - z = 3 \\ 2x - 5y + z = 0 \\ 3x - 3y + 2z = -1 \end{cases}$

$\begin{cases} x - 4y - z = 3 \\ 3y + 3z = -6 \\ 9y + 5z = -10 \end{cases}$

$\begin{cases} x - 4y - z = 3 \\ y + z = -2 \\ -4z = 8 \end{cases}$

$z = -2$

$y = -2 - (-2) = 0$

$x = 3 + 4(0) + (-2) = 1$

Answer: $(1, 0, -2)$

7. $6 = a(0)^2 + b(0) + c \implies c = 6$

$2 = a(-2)^2 + b(-2) + c$

$\frac{9}{2} = a(3)^2 + b(3) + c$

Hence, $\begin{cases} 4a - 2b + 6 = 2 \text{ or } 2a - b = -2 \\ 9a + 3b + 6 = \frac{9}{2} \text{ or } 9a + 3b = -\frac{3}{2}. \end{cases}$

Solving this system for a and b, you obtain

$a = -\frac{1}{2}, b = 1$. Thus, $y = -\frac{1}{2}x^2 + x + 6$.

8. $\dfrac{5x-2}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$

$5x - 2 = A(x-1) + B = Ax + (-A + B)$

$\begin{cases} A = 5 \\ -A + B = -2 \implies B = 3 \end{cases}$

$\dfrac{5x-2}{(x-1)^2} = \dfrac{5}{x-1} + \dfrac{3}{(x-1)^2}$

9. $\dfrac{x^3 + x^2 + x + 2}{x^4 + x^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{Cx + D}{x^2 + 1}$

$$x^3 + x^2 + x + 2 = Ax(x^2 + 1) + B(x^2 + 1) + (Cx + D)x^2$$
$$= (A + C)x^3 + (B + D)x^2 + Ax + B$$

$\begin{cases} A + C = 1 \\ B + D = 1 \\ A = 1 \\ B = 2 \end{cases}$

$A = 1, C = 0, B = 2, D = -1$

$\dfrac{x^3 + x^2 + x + 2}{x^4 + x^2} = \dfrac{1}{x} + \dfrac{2}{x^2} - \dfrac{1}{x^2 + 1}$

10. $\begin{bmatrix} 2 & 1 & 2 & \vdots & 4 \\ 2 & 2 & 0 & \vdots & 5 \\ 2 & -1 & 6 & \vdots & 2 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 2 & \vdots & 1.5 \\ 0 & 1 & -2 & \vdots & 1 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$.

Infinite number of solutions. Let $z = a$, $y = 2a + 1$, $x = 1.5 - 2a$.

Answer: $(1.5 - 2a, 1 + 2a, a)$, where a is any real number

11. $\begin{bmatrix} 2 & 3 & 1 & \vdots & 10 \\ 2 & -3 & -3 & \vdots & 22 \\ 4 & -2 & 3 & \vdots & -2 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & 0 & \vdots & 5 \\ 0 & 1 & 0 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & -6 \end{bmatrix}$.

Answer: $(5, 2, -6)$

12. (a) $A - B = \begin{bmatrix} 1 & 0 & 4 \\ -7 & -6 & -1 \\ 0 & 4 & 0 \end{bmatrix}$

(b) $3A = \begin{bmatrix} 15 & 12 & 12 \\ -12 & -12 & 0 \\ 3 & 6 & 0 \end{bmatrix}$

(c) $3A - 2B = \begin{bmatrix} 7 & 4 & 12 \\ -18 & -16 & -2 \\ 1 & 10 & 0 \end{bmatrix}$

(d) $AB = \begin{bmatrix} 36 & 20 & 4 \\ -28 & -24 & -4 \\ 10 & 8 & 2 \end{bmatrix}$

13. $\begin{bmatrix} -2 & 2 & 3 & \vdots & 1 & 0 & 0 \\ 1 & -1 & 0 & \vdots & 0 & 1 & 0 \\ 0 & 1 & 4 & \vdots & 0 & 0 & 1 \end{bmatrix}$

reduces to

$\begin{bmatrix} 1 & 0 & 0 & \vdots & -\frac{4}{3} & -\frac{5}{3} & 1 \\ 0 & 1 & 0 & \vdots & -\frac{4}{3} & -\frac{8}{3} & 1 \\ 0 & 0 & 1 & \vdots & \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$.

$$A^{-1} = \begin{bmatrix} -\frac{4}{3} & -\frac{5}{3} & 1 \\ -\frac{4}{3} & -\frac{8}{3} & 1 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$$

$$A^{-1}\begin{bmatrix} 7 \\ -5 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}.$$

Answer: $(-2, 3, -1)$

14. $\begin{vmatrix} -25 & 18 \\ 6 & -7 \end{vmatrix} = (-25)(-7) - 6(18) = 67$

15. $\det(A) = \begin{vmatrix} 4 & 0 & 3 \\ 1 & -8 & 2 \\ 3 & 2 & 2 \end{vmatrix} = 4(-16 - 4) - 0 + 3(2 + 24)$

$= -80 + 78 = -2$

16. $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 1 & 1 \\ 4 & 11 & 1 \\ -1 & -5 & 1 \end{vmatrix}$

$= -1(16) - 1(4 + 1) + 1(-20 + 11)$

$= -16 - 5 - 9 = -30$

Area $= |-30| = 30$ square units

17. $x = \dfrac{\begin{vmatrix} 3 & -2 \\ -1 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & -2 \\ 1 & 4 \end{vmatrix}} = \dfrac{10}{10} = 1$

$y = \dfrac{\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -2 \\ 1 & 4 \end{vmatrix}} = \dfrac{-5}{10} = -\dfrac{1}{2}$

Answer: $\left(1, -\dfrac{1}{2}\right)$

18. Upper left: $400 + x_2 = x_1$

Upper right: $x_1 + x_3 = x_4 + 600$

Lower left: $300 = x_2 + x_3 + x_5$

Lower right: $x_5 + x_4 = 100$

$$\begin{cases} x_1 - x_2 & = 400 \\ x_1 + x_3 - x_4 & = 600 \\ x_2 + x_3 + x_5 & = 300 \\ x_4 + x_5 & = 100 \end{cases}$$

Solving this system:

$$\left[\begin{array}{rrrrr|r} 1 & -1 & 0 & 0 & 0 & 400 \\ 1 & 0 & 1 & -1 & 0 & 600 \\ 0 & 1 & 1 & 0 & 1 & 300 \\ 0 & 0 & 0 & 1 & 1 & 100 \end{array}\right] \rightarrow \left[\begin{array}{rrrrr|r} 1 & 0 & 1 & 0 & 1 & 700 \\ 0 & 1 & 1 & 0 & 1 & 300 \\ 0 & 0 & 0 & 1 & 1 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Letting $x_3 = a$ and $x_5 = b$ be real numbers, we have:

$x_5 = b$

$x_4 = 100 - b$

$x_3 = a$

$x_2 = 300 - a - b$

$x_1 = 700 - b - a$

Chapter 8 Chapter Test Solutions

1. $a_n = \left(-\frac{2}{3}\right)^{n-1}$

$a_1 = \left(-\frac{2}{3}\right)^{1-1} = \left(-\frac{2}{3}\right)^0 = 1$

$a_2 = -\frac{2}{3}$

$a_3 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$

$a_4 = \left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$

$a_5 = \left(-\frac{2}{3}\right)^4 = \frac{16}{81}$

2. $a_1 = 12,\ a_{k+1} = a_k + 4$

$a_2 = 12 + 4 = 16$

$a_3 = 16 + 4 = 20$

$a_4 = 20 + 4 = 24$

$a_5 = 24 + 4 = 28$

3. $b_1 = -x$

$b_2 = \dfrac{x^2}{2}$

$b_3 = -\dfrac{x^3}{3}$

$b_4 = \dfrac{x^4}{4}$

$b_5 = -\dfrac{x^5}{5}$

4. $b_1 = -\dfrac{x^3}{3!} = -\dfrac{x^3}{6}$

$b_2 = -\dfrac{x^5}{5!} = -\dfrac{x^5}{120}$

$b_3 = -\dfrac{x^7}{7!}$

$b_4 = -\dfrac{x^9}{9!}$

$b_5 = -\dfrac{x^{11}}{11!}$

5. $\dfrac{11!4!}{4!7!} = \dfrac{11!}{7!}$

$= \dfrac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!}$

$= 11 \cdot 10 \cdot 9 \cdot 8$

$= 7920$

6. $\dfrac{n!}{(n+1)!} = \dfrac{n!}{(n+1)n!} = \dfrac{1}{n+1}$

7. $\dfrac{2n!}{(n-1)!} = \dfrac{2n(n-1)!}{(n-1)!} = 2n$

8. $a_n = n^2 + 1,\ n = 1, 2, 3, \ldots$

9. $a_n = dn + c$

$c = a_1 - d = 5000 - (-100) = 5100 \implies$

$a_n = -100n + 5100 = 5000 - 100(n-1)$

10. $a_n = a_1 r^{n-1},\ a_1 = 4,\ r = \frac{1}{2} \implies a_n = 4\left(\frac{1}{2}\right)^{n-1}$

11. $\displaystyle\sum_{n=1}^{12} \frac{2}{3n+1}$

12. $\displaystyle 2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots = \sum_{n=1}^{\infty} 2\left(\frac{1}{4}\right)^{n-1}$

$\displaystyle = \sum_{n=0}^{\infty} \frac{1}{2^{2n-1}}$

13. $\displaystyle\sum_{n=1}^{7} (8n - 5) = 8\left(\frac{7(8)}{2}\right) - 5(7) = 224 - 35 = 189$

14. $\displaystyle\sum_{n=1}^{8} 24\left(\frac{1}{6}\right)^{n-1} = 24\left(\frac{1-(1/6)^8}{1-(1/6)}\right)$

$\displaystyle = 24\left(\frac{6}{5}\right)\left(1 - \left(\frac{1}{6}\right)^8\right)$

$\approx 28.79998 \approx 28.80$

15. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{5^{n-1}} = \sum_{n=0}^{\infty} 5\left(\frac{(-1)2}{5}\right)^n$

$\displaystyle = 5\left(\frac{1}{1-(-2/5)}\right)$

$\displaystyle = 5\left(\frac{5}{7}\right)$

$\displaystyle = \frac{25}{7}$

16. (1) For $n = 1$, $3 = \dfrac{3(1)(1+1)}{2}$.

(2) Assume $S_k = 3 + 6 + \cdots + 3k = \dfrac{3k(k+1)}{2}$.

Then $S_{k+1} = 3 + 6 + \cdots + 3k + 3(k+1)$

$= S_k + 3(k+1)$

$= \dfrac{3k(k+1)}{2} + 3(k+1)$

$= \dfrac{k+1}{2}[3k + 6]$

$= \dfrac{3(k+1)(k+2)}{2}$.

Therefore, the formula is true for all positive integers n.

17. $(2a - 5b)^4 = (2a)^4 - 4(2a)^3(5b) + 6(2a)^2(5b)^2 - 4(2a)(5b)^3 + (5b)^4$

$= 16a^4 - 160a^3b + 600a^2b^2 - 1000ab^3 + 625b^4$

18. ${}_9C_3 = 84$

19. ${}_{20}C_3 = 1140$

20. ${}_9P_2 = \dfrac{9!}{7!} = 9 \cdot 8 = 72$

21. ${}_{70}P_3 = \dfrac{70!}{67!}$

$= 70 \cdot 69 \cdot 68$

$= 328{,}440$

22. $4 \cdot {}_nP_3 = {}_{n+1}P_4$

$4\dfrac{n!}{(n-3)!} = \dfrac{(n+1)!}{(n-3)!}$

$4n! = (n+1)!$

$n = 3$

23. $26 \cdot 10 \cdot 10 \cdot 10 = 26{,}000$ ways

24. ${}_{25}C_4 = \dfrac{25!}{21!\,4!} = \dfrac{25 \cdot 24 \cdot 23 \cdot 22}{24} = 12{,}650$ ways

25. There are six red face cards $\Longrightarrow$

probability $= \dfrac{6}{52} = \dfrac{3}{26}$.

26. $\dfrac{1}{{}_{11}C_6} = \dfrac{1}{462}$

27. (a) $\left(\frac{30}{60}\right)\left(\frac{30}{60}\right) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

(b) $\frac{11}{60} \cdot \frac{11}{60} = \frac{121}{3600} \approx 0.0336$

(c) $\frac{1}{60} \approx 0.0167$

28. $1 - 0.75 = 0.25 = 25\%$

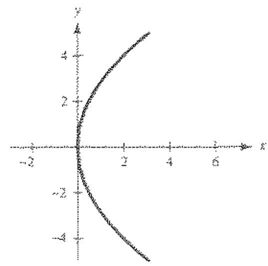
Chapter 9 Chapter Test Solutions

1. $y^2 = 8x = 2(4)x$

Parabola

Vertex: $(0, 0)$

Focus: $(2, 0)$



2. $y^2 - 4x + 4 = 0$

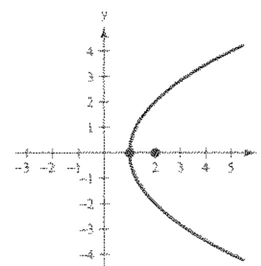
$y^2 = 4x - 4$

$y^2 = 4(x - 1)$

Parabola

Vertex: $(1, 0)$

Focus: $(2, 0)$



3. $x^2 - 4y^2 - 4x = 0$

$x^2 - 4x + 4 - 4y^2 = 4$

$(x - 2)^2 - 4y^2 = 4$

$\dfrac{(x - 2)^2}{4} - y^2 = 1$

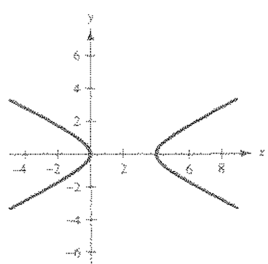
Hyperbola

Center: $(2, 0)$

$a = 2, b = 1, c = \sqrt{5}$

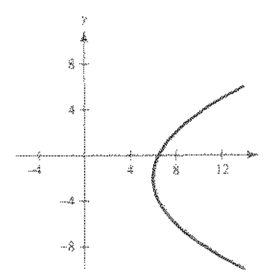
Vertices: $(0, 0), (4, 0)$

Foci: $\left(2 \pm \sqrt{5}, 0\right)$



4. Vertex: $(6, -2), p = 2$

$(y + 2)^2 = 4(2)(x - 6)$



5. Center: $(-6, 3)$

$a = 7, b = 4$

$\dfrac{(x + 6)^2}{16} + \dfrac{(y - 3)^2}{49} = 1$

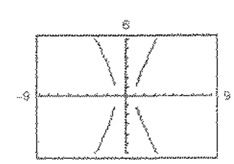
6. $a = 3, \dfrac{3}{2} = \dfrac{a}{b} \Longrightarrow b = 2$

$\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$

7. $x^2 - \dfrac{y^2}{4} = 1$

$\dfrac{y^2}{4} = x^2 - 1$

$y = \pm 2\sqrt{x^2 - 1}$

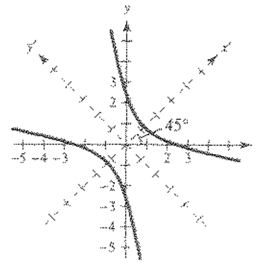


8. (a) $\cot 2\theta = \dfrac{A - C}{B} = \dfrac{1 - 1}{6} = 0 \implies \theta = \dfrac{\pi}{4}$ or $45°$

(b) $B^2 - 4AC = 36 - 4 = 32 > 0 \implies$ Hyperbola

$y^2 + 6xy + (x^2 - 6) = 0$

$$y = \frac{-6x \pm \sqrt{36x^2 - 4(x^2 - 6)}}{2}$$



9. $x^2 + 2y^2 - 4x + 6y - 5 = 0$

$x + y + 5 = 0$

$y = -x - 5$:

$x^2 + 2(-x - 5)^2 - 4x + 6(-x - 5) - 5 = 0$

$x^2 + 2x^2 + 20x + 50 - 4x - 6x - 30 - 5 = 0$

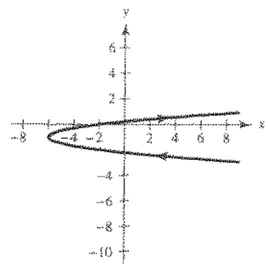
$3x^2 + 10x + 15 = 0$

This quadratic has no real solutions. Therefore, no solution.

10. $x = t^2 - 6$

$y = \frac{1}{2}t - 1 \implies t = 2(y + 1)$

$x = [2(y + 1)]^2 - 6 = 4y^2 + 8y - 2$



Parabola

$x = 4y^2 + 8y - 2$ or $(y + 1)^2 = \frac{1}{4}(x + 6)$

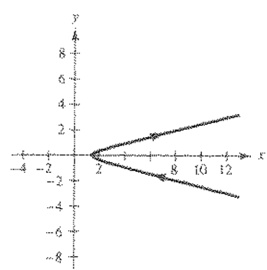
11. $x = \sqrt{t^2 + 2}$

$y = \dfrac{t}{4} \implies t = 4y$

$x = \sqrt{16y^2 + 2}$

$x^2 = 16y^2 + 2$

Right-hand portion of hyperbola



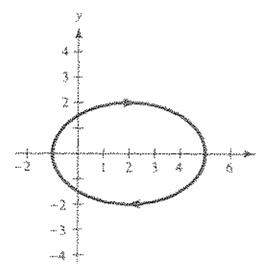
12. $x = 2 + 3\cos\theta$

$y = 2\sin\theta$

$\left(\dfrac{x - 2}{3}\right)^2 + \left(\dfrac{y}{2}\right)^2 = 1$

$\dfrac{(x - 2)^2}{9} + \dfrac{y^2}{4} = 1$

Ellipse



13. $x = t, \quad y = t^2 + 10$

$x = -t, \quad y = t^2 + 10$

14. Sample answers:

$x = 4 - t^2$

$y = t$

$x = 4 - 4t^2$

$y = 2t$

15. Sample answers:

$x = \pm\sqrt{16 - 4t^2}$

$y = t$

$x = \pm\sqrt{16 - t^2}$

$y = \frac{1}{2}t$

16. $(r, \theta) = \left(-14, \dfrac{5\pi}{3}\right) \implies (x, y) = \left(-7, 7\sqrt{3}\right)$

17. $(x, y) = (2, -2),\ r = \sqrt{8} = 2\sqrt{2},\ \theta = \dfrac{7\pi}{4}$

$$(r, \theta) = \left(2\sqrt{2}, \frac{7\pi}{4}\right) = \left(2\sqrt{2}, -\frac{\pi}{4}\right) = \left(-2\sqrt{2}, \frac{3\pi}{4}\right)$$

18. $x^2 + y^2 - 12y = 0$

$r^2 - 12\,r \sin\theta = 0$

$r^2 = 12\,r\sin\theta$

$r = 12\sin\theta$

19. $r = 2\sin\theta$

$r^2 = 2r\sin\theta$

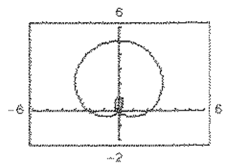
$x^2 + y^2 = 2y$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 + (y-1)^2 = 1$, Circle

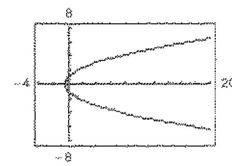
20. $r = 2 + 3\sin\theta$

Limaçon with inner loop



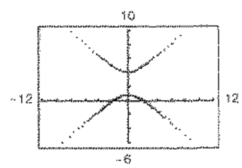
21. $r = \dfrac{1}{1 - \cos\theta}$

$e = 1 \Longrightarrow$ Parabola



22. $r = \dfrac{4}{2 + 3\sin\theta} = \dfrac{2}{1 + \frac{3}{2}\sin\theta}$

$e = \dfrac{3}{2} \Longrightarrow$ Hyperbola



23. $r = \dfrac{ep}{1 + e\sin\theta} = \dfrac{(1/4)(4)}{1 + (1/4)\sin\theta}$

$r = \dfrac{4}{4 + \sin\theta} = \dfrac{1}{1 + (1/4)\sin\theta}$

24. $r = \dfrac{ep}{1 + e\sin\theta} = \dfrac{(5/4)(2)}{1 + (5/4)\sin\theta} = \dfrac{10}{4 + 5\sin\theta}$

25. $r = 8\cos 3\theta$

The maximum value of $|r|$ occurs when $|\cos 3\theta| = 1$. Hence, the maximum is $8 = |r|$.

$r = 0 \Longrightarrow \cos 3\theta = 0$

$\Longrightarrow 3\theta = \dfrac{\pi}{2} + n\pi$

$\Longrightarrow \theta = \dfrac{\pi}{6} + \dfrac{n\pi}{3}$

On the interval $0 \le \theta \le \pi$, $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}$.

Chapters 7–9 Cumulative Test Solutions

1. $\begin{bmatrix} -1 & -3 & \vdots & 5 \\ 4 & 2 & \vdots & 10 \end{bmatrix}$ row reduces to $\begin{bmatrix} 1 & 0 & \vdots & 4 \\ 0 & 1 & \vdots & -3 \end{bmatrix}$.

Answer: $(4, -3)$

2. $2x - y^2 = 0$

$x - y = 4 \implies x = y + 4$

$2(y+4) - y^2 = 0$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y = 4,\ x = 8$

$y = -2,\ x = 2$

Solutions: $(2, -2), (8, 4)$

3. $\begin{bmatrix} 2 & -3 & 1 & \vdots & 13 \\ -4 & 1 & -2 & \vdots & -6 \\ 1 & -3 & 3 & \vdots & 12 \end{bmatrix}$ row reduces to

$\begin{bmatrix} 1 & 0 & 0 & \vdots & \frac{3}{5} \\ 0 & 1 & 0 & \vdots & -4 \\ 0 & 0 & 1 & \vdots & -\frac{1}{5} \end{bmatrix}$.

Answer: $\left(\frac{3}{5}, -4, -\frac{1}{5}\right)$

4. $\begin{bmatrix} 1 & -4 & 3 & \vdots & 5 \\ 5 & 2 & -1 & \vdots & 1 \\ -2 & -8 & 0 & \vdots & 30 \end{bmatrix}$ row reduces to

$\begin{bmatrix} 1 & 0 & 0 & \vdots & 1 \\ 0 & 1 & 0 & \vdots & -4 \\ 0 & 0 & 1 & \vdots & -4 \end{bmatrix}$.

Solution: $(1, -4, -4)$

5. $3A - B = \begin{bmatrix} -7 & -10 & -16 \\ -6 & 18 & 9 \\ -12 & 16 & 7 \end{bmatrix}$

6. $5A + 3B = \begin{bmatrix} -18 & 15 & -14 \\ 28 & 11 & 34 \\ -20 & 52 & -1 \end{bmatrix}$

7. $AB = \begin{bmatrix} 3 & -31 & 2 \\ 22 & 18 & 6 \\ 52 & -40 & 14 \end{bmatrix}$

8. $BA = \begin{bmatrix} 5 & 36 & 31 \\ -36 & 12 & -36 \\ 16 & 0 & 18 \end{bmatrix}$

9. (a) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}^{-1} = \begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$

(b) $\det(A) = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{vmatrix} = 1(-105 - 70) - 2(-45 - 50) - 1(-21 + 35)$

$= -175 + 190 - 14 = 1$

10. $\begin{vmatrix} 0 & 0 & 1 \\ 6 & 2 & 1 \\ 8 & 10 & 1 \end{vmatrix} = 44 \implies \text{Area} = \frac{1}{2}(44) = 22$ sq. units

11. (a) $a_1 = \dfrac{(-1)^{1+1}}{2(1)+3} = \dfrac{1}{5}$

$a_2 = -\dfrac{1}{7}$

$a_3 = \dfrac{1}{9}$

$a_4 = -\dfrac{1}{11}$

$a_5 = \dfrac{1}{13}$

(b) $a_1 = 3(2)^{1-1} = 3$

$a_2 = 6$

$a_3 = 12$

$a_4 = 24$

$a_5 = 48$

12. $\displaystyle\sum_{k=1}^{6} (7k - 2) = \frac{7(6)(7)}{2} - 2(6) = 135$

13. $\displaystyle\sum_{k=1}^{4} \frac{2}{k^2+4} = \frac{2}{1+4} + \frac{2}{4+4} + \frac{2}{9+4} + \frac{2}{16+4}$

≈ 0.9038

14. $\displaystyle\sum_{n=0}^{10} 9\left(\frac{3}{4}\right)^n = 9\left(\frac{1-(3/4)^{11}}{1-(3/4)}\right) \approx 34.4795$

15. $\displaystyle\sum_{n=0}^{50} 100\left(-\frac{1}{2}\right)^n = \sum_{n=1}^{51} 100\left(-\frac{1}{2}\right)^{n-1}$

$\displaystyle = 100\frac{1-(-1/2)^{51}}{1-(-1/2)}$

$\displaystyle \approx \frac{2}{3}(100) \approx 66.67$

16. $\displaystyle\sum_{n=0}^{\infty} 3\left(-\frac{3}{5}\right)^n = \frac{3}{1-(-3/5)} = \frac{3}{8/5} = \frac{15}{8}$

17. $\displaystyle\sum_{n=1}^{\infty} 5(-0.02)^n = 5(-0.02)\frac{1}{1-(-0.02)}$

$\displaystyle = \frac{-0.1}{1.02} = \frac{-5}{51}$

18. $\displaystyle 4 - 2 + 1 - \frac{1}{2} + \frac{1}{4} - \cdots = \sum_{n=0}^{\infty} 4\left(-\frac{1}{2}\right)^n = \frac{4}{1-(-1/2)} = \frac{8}{3}$

19. For $n = 1$, $3 = 1(2 + 1)$ and the formula is true. Assume true for k, and consider

$$\begin{aligned} 3 + 7 + \cdots + (4k - 1) + (4(k + 1) - 1) &= 3 + 7 + \cdots + (4k - 1) + (4k + 3) \\ &= k(2k + 1) + (4k + 3) \\ &= 2k^2 + 5k + 3 \\ &= (k + 1)(2k + 3) \\ &= (k + 1)(2(k + 1) + 1) \end{aligned}$$

which shows that the formula is true for $k + 1$.

20. $(x + 3)^4 = x^4 + 12x^3 + 54x^2 + 108x + 81$

21. $(2x + y^2)^5 = 32x^5 + 80x^4y^2 + 80x^3y^4 + 40x^2y^6 + 10xy^8 + y^{10}$

22. $(x - 2y)^6 = x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6$

23. $(3a - 4b)^8 = 6561a^8 - 69{,}984a^7b + 326{,}592a^6b^2 - 870{,}912a^5b^3 + 1{,}451{,}520a^4b^4 - 1{,}548{,}288a^3b^5$
$+ 1{,}032{,}192a^2b^6 - 393{,}216ab^7 + 65{,}536b^8$

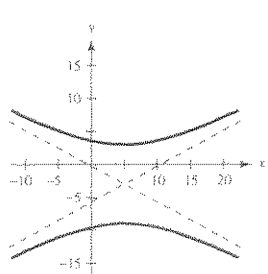
24. $\dfrac{5!}{2!2!1!} = \dfrac{120}{4} = 30$

25. $\dfrac{6!}{3!} = 6 \cdot 5 \cdot 4 = 120$

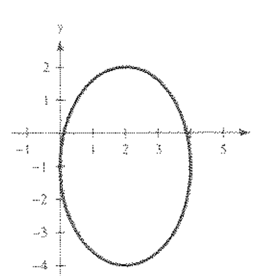
26. $\dfrac{10!}{2!2!2!} = 453{,}600$

27. $\dfrac{10!}{3!2!2!} = 151{,}200$

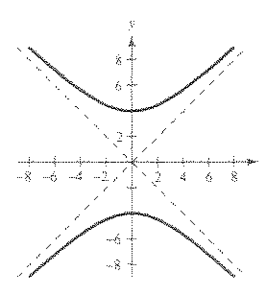
28. Hyperbola with center $(5, -3)$



29. Ellipse with center $(2, -1)$



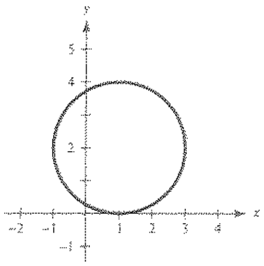
30. Hyperbola with center $(0, 0)$



31. $(x^2 - 2x + 1) + (y^2 - 4y + 4) = -1 + 1 + 4$

$$(x - 1)^2 + (y - 2)^2 = 4$$

Circle



32. $(x - 2)^2 = 4p(y - 3)$

$(0, 0)$: $4 = 4p(-3) \implies p = -\frac{1}{3}$

$(x - 2)^2 = 4\left(-\frac{1}{3}\right)(y - 3)$

$(x - 2)^2 = -\frac{4}{3}(y - 3)$

33. Center: $(1, 4)$

$a = 5, b = 2$

$$\frac{(x - 1)^2}{25} + \frac{(y - 4)^2}{4} = 1$$

34. Center: $(0, -4)$; $a = 2$

$$\frac{(y + 4)^2}{4} - \frac{x^2}{b^2} = 1$$

$(4, 0)$: $4 - \dfrac{16}{b^2} = 1 \implies \dfrac{16}{b^2} = 3 \implies b^2 = \dfrac{16}{3}$

$$\frac{(y + 4)^2}{4} - \frac{x^2}{16/3} = 1$$

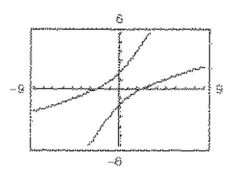
35. $B^2 - 4AC = 16 - 8 = 8 \implies$ Hyperbola

$\cot 2\theta = \dfrac{1 - 2}{-4} = \dfrac{1}{4} \implies \theta \approx 38°$

Graph as:

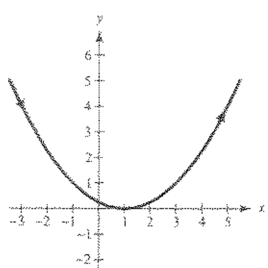
$2y^2 - 4xy + (x^2 - 6) = 0$

$$y = \frac{4x \pm \sqrt{16x^2 - 8(x^2 - 6)}}{4}$$



36. $x = 2t + 1, y = t^2$

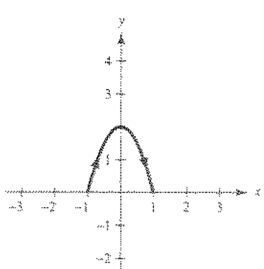
(a), (b)



(c) $t = \dfrac{x - 1}{2} \implies y = \left(\dfrac{x - 1}{2}\right)^2 = \dfrac{1}{4}(x - 1)^2$

37. $x = \cos\theta, y = 2\sin^2\theta$

(a), (b)



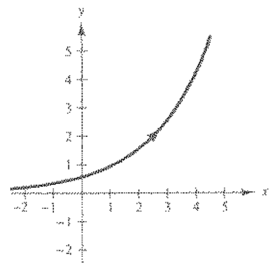
(c) $y = 2\sin^2\theta$

$= 2(1 - \cos^2\theta)$

$= 2(1 - x^2),\ -1 \le x \le 1$

38. $x = 4 \ln t,\ y = \frac{1}{2}t^2$

(a), (b)



(c) $t = e^{x/4} \Longrightarrow y = \frac{1}{2}e^{x/2}$

39. $y = 3x - 2$

Sample answers:

$x = t$

$y = 3t - 2$

$x = -t$

$y = -3t - 2$

40. Sample answers:

$x = \pm\sqrt{t^2 + 16}$

$y = t$

$x = t$

$y = \pm\sqrt{t^2 - 16}$

41. $y = \dfrac{2}{x}$

Sample answers:

$x = t$

$y = \dfrac{2}{t}$

$x = -t$

$y = -\dfrac{2}{t}$

42. Sample answers:

$x = t$

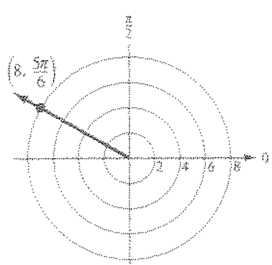
$y = \dfrac{e^{2t}}{e^{2t} + 1}$

$x = \dfrac{1}{2}t$

$y = \dfrac{e^t}{e^t + 1}$

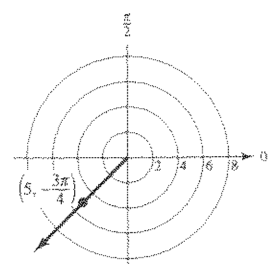
43. $(r, \theta) = \left(8, \dfrac{5\pi}{6}\right)$

$\left(8, -\dfrac{7\pi}{6}\right), \left(-8, -\dfrac{\pi}{6}\right), \left(-8, \dfrac{11\pi}{6}\right)$



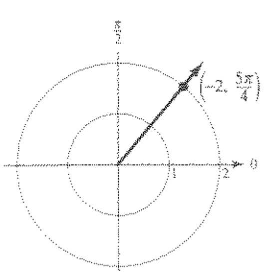
44. $(r, \theta) = \left(5, -\dfrac{3\pi}{4}\right)$

$\left(5, \dfrac{5\pi}{4}\right), \left(-5, \dfrac{\pi}{4}\right), \left(-5, -\dfrac{7\pi}{4}\right)$



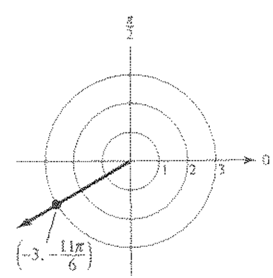
45. $(r, \theta) = \left(-2, \dfrac{5\pi}{4}\right)$

$\left(-2, -\dfrac{3\pi}{4}\right), \left(2, \dfrac{\pi}{4}\right), \left(2, -\dfrac{7\pi}{4}\right)$



46. $(r, \theta) = \left(-3, -\dfrac{11\pi}{6}\right)$

$\left(-3, \dfrac{\pi}{6}\right), \left(3, \dfrac{7\pi}{6}\right), \left(3, -\dfrac{5\pi}{6}\right)$

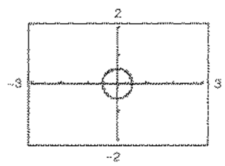


47. $$4x + 4y + 1 = 0$$
$$4r\cos\theta + 4r\sin\theta + 1 = 0$$
$$r[4\cos\theta + 4\sin\theta] = -1$$
$$r = \frac{-1}{4\cos\theta + 4\sin\theta}$$

48. $r = 2\cos\theta$
$$r^2 = 2r\cos\theta$$
$$x^2 + y^2 = 2x$$
$$x^2 - 2x + 1 + y^2 = 1$$
$$(x-1)^2 + y^2 = 1, \text{ Circle}$$

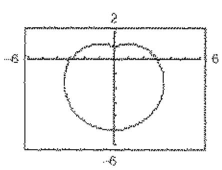
49. $$r = \frac{2}{4 - 5\cos\theta}$$
$$4r - 5r\cos\theta = 2$$
$$4(x^2 + y^2)^{1/2} - 5x = 2$$
$$16(x^2 + y^2) = (5x + 2)^2 = 25x^2 + 20x + 4$$
$$16y^2 - 9x^2 - 20x = 4$$

50. $r = -\frac{\pi}{6}$, Circle



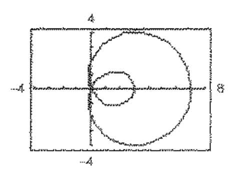
51. $r = 3 - 2\sin\theta$

Limaçon



52. $r = 2 + 5\cos\theta$

Limaçon



53. $$32{,}500 + 32{,}500(1.05) + \cdots + 32{,}500(1.05)^{14} = \sum_{n=1}^{15} 32{,}500(1.05)^{n-1}$$
$$= 32{,}500\left(\frac{1 - 1.05^{15}}{1 - 1.05}\right) \approx \$701{,}303.32$$

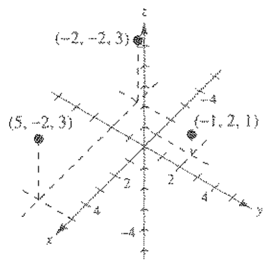
54. There are two ways to select the first digit (4 or 5), and two ways for the second digit. Hence, $p = \frac{1}{4}$.

55. Let $y = -ax^2 + 16$.

$(6, 14)$: $14 = -a(6)^2 + 16$
$$36a = 2$$
$$a = \frac{1}{18}$$
$$y = -\frac{1}{18}x^2 + 16$$
$y = 0$: $16 = \frac{1}{18}x^2$
$$x^2 = 288$$
$$x = 12\sqrt{2}$$
Width: $24\sqrt{2}$ meters

Chapter 10 Chapter Test Solutions

1.



2. $AB = \sqrt{(8-6)^2 + (-2-4)^2 + (5+1)^2} = \sqrt{76}$

$AC = \sqrt{(8+4)^2 + (-2-3)^2 + (5-0)^2} = \sqrt{144 + 25 + 25} = \sqrt{194}$

$BC = \sqrt{(6+4)^2 + (4-3)^2 + (-1-0)^2} = \sqrt{100 + 1 + 1} = \sqrt{102}$

No. $\left(\sqrt{76}\right)^2 + \left(\sqrt{102}\right)^2 \neq \left(\sqrt{194}\right)^2$

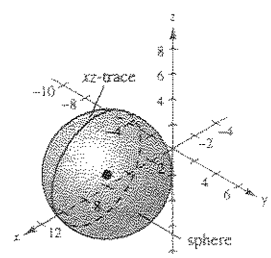
3. Midpoint $= \left(\dfrac{8+6}{2}, \dfrac{-2+4}{2}, \dfrac{5-1}{2}\right) = (7, 1, 2)$

4. Diameter $= \sqrt{(8-6)^2 + (-2-4)^2 + (5+1)^2}$

$= \sqrt{4 + 36 + 36} = \sqrt{76}$

Radius $= \sqrt{19}$

$(x-7)^2 + (y-1)^2 + (z-2)^2 = 19$



5. $\mathbf{v} = \langle 4-2, 4-(-1), -7-3\rangle = \langle 2, 5, -10\rangle$

$\|\mathbf{v}\| = \sqrt{2^2 + 5^2 + (-10)^2} = \sqrt{129}$

6. $\mathbf{v} = \langle 3-6, -3-2, 8-0\rangle = \langle -3, -5, 8\rangle$

$\|\mathbf{v}\| = \sqrt{(-3)^2 + (-5)^2 + 8^2} = \sqrt{98} = 7\sqrt{2}$

7. $\mathbf{u} = \langle 6-8, 4-(-2), -1-5\rangle = \langle -2, 6, -6\rangle$

$\mathbf{v} = \langle -4-8, 3-(-2), 0-5\rangle = \langle -12, 5, -5\rangle$

8. (a) $\|\mathbf{v}\| = \sqrt{(-12)^2 + 5^2 + (-5)^2} = \sqrt{194}$

(b) $\mathbf{u}\cdot\mathbf{v} = (-2)(-12) + 6(5) + (-6)(-5) = 84$

(c) $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 6 & -6 \\ -12 & 5 & -5 \end{vmatrix} = \langle 0, 62, 62\rangle$

9. $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \dfrac{84}{\sqrt{76}\sqrt{194}} \approx 0.6918 \implies \theta \approx 46.23°$ or 0.8068 radians

10. (a) $x = 8 - 2t, y = -2 + 6t, z = 5 - 6t$

(b) $\dfrac{x-8}{-2} = \dfrac{y+2}{6} = \dfrac{z-5}{-6}$

11. $\mathbf{u}\cdot\mathbf{v} = 0 - 2 - 6 \neq 0$ and $\mathbf{u} \neq c\mathbf{v} \implies$ neither

12. $\mathbf{u}\cdot\mathbf{v} = -2 + 3 - 1 = 0 \implies$ orthogonal

13. First two points: $\mathbf{v} = \langle 4, 8, -2 \rangle$

Last two points: $\mathbf{w} = \langle 4, 8, -2 \rangle$

Opposite sides are parallel and equal length.

Adjacent sides: $\mathbf{v}$ and $\mathbf{u} = \langle 1, -3, 3 \rangle$

Area $= \|\mathbf{u} \times \mathbf{v}\|$

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 3 \\ 4 & 8 & -2 \end{vmatrix} = \langle -18, 14, 20 \rangle$$

$$\|\mathbf{u} \times \mathbf{v}\| = \sqrt{18^2 + 14^2 + 20^2} = 2\sqrt{230} \approx 30.33 \text{ square units}$$

14. $\mathbf{u} = \langle 0, 8, -1 \rangle, \mathbf{v} = \langle 4, 5, -4 \rangle$

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & -1 \\ 4 & 5 & -4 \end{vmatrix} = \langle -27, -4, -32 \rangle$$

$$\begin{aligned} \text{Plane: } -27(x + 3) - 4(y + 4) - 32(z - 2) &= 0 \\ -27x - 4y - 32z - 33 &= 0 \\ 27x + 4y + 32z + 33 &= 0 \end{aligned}$$

15. Let $A(0, 0, 5)$ be the vertex.

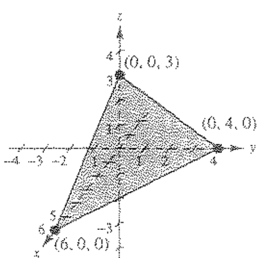
$\mathbf{u} = \overrightarrow{AD} = \langle 4, 0, 0 \rangle$, $\mathbf{v} = \overrightarrow{AB} = \langle 0, 10, 0 \rangle$,

$\mathbf{w} = \overrightarrow{AE} = \langle 0, 1, -5 \rangle$

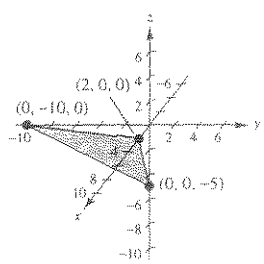
$$\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = \begin{vmatrix} 4 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 1 & -5 \end{vmatrix} = 4(-50) = -200$$

Volume $= |-200| = 200$ cubic units

16. $2x + 3y + 4z = 12$



17. $5x - y - 2z = 10$

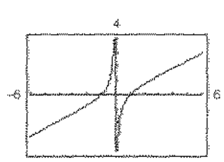


18. $\mathbf{n} = \langle 3, -2, 1 \rangle$, $Q = (2, -1, 6)$, $P = (0, 0, 6)$ in plane, $\overrightarrow{PQ} = \langle 2, -1, 0 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{|8|}{\sqrt{14}} = \frac{4\sqrt{14}}{7}$$

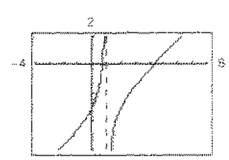
Chapter 11 Chapter Test Solutions

1. $\displaystyle\lim_{x \to -2} \frac{x^2 - 1}{2x} = \frac{(-2)^2 - 1}{2(-2)} = -\frac{3}{4}$



Limit is -0.75.

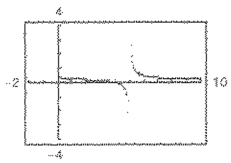
2.



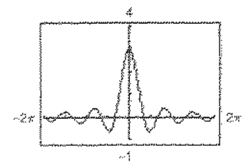
The limit does not exist.

$\displaystyle\lim_{x \to 1} \frac{-x^2 + 5x - 3}{1 - x}$ does not exist.

$x = 1$ is a vertical asymptote.

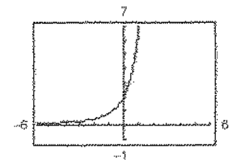
3. 

$\lim_{x \to 5} \frac{\sqrt{x} - 2}{x - 5}$ does not exist.

4. 

$\lim_{x \to 0} \frac{\sin 3x}{x} = 3$

$f(x) = \frac{\sin 3x}{x}$

5. 

$\lim_{x \to 0} \frac{e^{2x} - 1}{x} = 2$

$f(x) = \frac{e^{2x} - 1}{x}$

6. (a) $\frac{f(x + h) - f(x)}{h} = \frac{3(x + h)^2 - 5(x + h) - 2 - (3x^2 - 5x - 2)}{h}$

$= \frac{3x^2 + 6xh + 3h^2 - 5h - 3x^2}{h}$

$= 6x + 3h - 5$

$f'(x) = \lim_{h \to 0} [6x + 3h - 5] = 6x - 5$

$f'(2) = 6(2) - 5 = 7$

(b) $\frac{f(x + h) - f(x)}{h} = \frac{[2(x + h)^3 + 6(x + h)] - [2x^3 + 6x]}{h}$

$= \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 6x + 6h - 2x^3 - 6x}{h}$

$= \frac{6x^2h + 6xh^2 + 2h^3 + 6h}{h}$

$= 6x^2 + 6xh + 2h^2 + 6, \ h \neq 0$

$f'(x) = \lim_{h \to 0} [6x^2 + 6xh + 2h^2 + 6] = 6x^2 + 6$

$f'(-1) = 6(-1)^2 + 6 = 12$

7. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{5 - (2/5)(x + h) - (5 - (2/5)x)}{h}$

$= \lim_{h \to 0} \frac{-(2/5)h}{h} = -\frac{2}{5}$

8. $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x + h)^2 + 4(x + h) - 1 - [2x^2 + 4x - 1]}{h}$

$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 + 4h - 2x^2}{h}$

$= \lim_{h \to 0} (4x + 2h + 4) = 4x + 4$

9. $$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{\dfrac{1}{x+3+h}-\dfrac{1}{x+3}}{h}$$
$$= \lim_{h\to 0}\frac{(x+3)-(x+3+h)}{h(x+3+h)(x+3)}$$
$$= \lim_{h\to 0}\frac{-1}{(x+3+h)(x+3)}$$
$$= \frac{-1}{(x+3)^2}$$

10. $\lim_{x\to\infty}\dfrac{6}{5x-1} = 0$

11. $\lim_{x\to\infty}\dfrac{1-3x^2}{x^2-5} = -3$

12. $\lim_{x\to-\infty}\dfrac{x^2}{3x+2}$ does not exist.

$f(x) = \dfrac{x^2}{3x+2}$ decreases without bound as $x \to -\infty$.

13. $0, \frac{3}{4}, \frac{14}{19}, \frac{12}{17}, \frac{36}{53}$

$\lim_{n\to\infty} a_n = \frac{1}{2}$

14. $0, 1, 0, \frac{1}{2}, 0$

$\lim_{n\to\infty} a_n = 0$

15. Width of each rectangle: $\frac{1}{2}$

Heights: $8, \frac{15}{2}, 6, \frac{7}{2}$

Area $\approx \frac{1}{2}\left[8 + \frac{15}{2} + 6 + \frac{7}{2}\right] = \frac{25}{2}$

16. Width: $\dfrac{4}{n}$, Height: $f\left(-2+\dfrac{4i}{n}\right) = \left(-2+\dfrac{4i}{n}\right) + 2 = \dfrac{4i}{n}$

$$A \approx \sum_{i=1}^{n}\left(\frac{4i}{n}\right)\left(\frac{4}{n}\right) = \frac{16}{n^2}\sum_{i=1}^{n} i = \frac{16}{n^2}\frac{n(n+1)}{2}$$

$$A = \lim_{n\to\infty}\frac{16}{n^2}\cdot\frac{n(n+1)}{2} = 8$$

17. $f(x) = 3 - x^2$, $[-1, 1]$

The width of each rectangle is $\dfrac{2}{n}$. The height is

$$f\left(-1+\frac{2i}{n}\right) = 3 - \left[-1+\frac{2i}{n}\right]^2$$
$$= 2 + \frac{4}{n}i - \frac{4}{n^2}i^2.$$

$$A \approx \sum_{i=1}^{n}\left[2+\frac{4}{n}i-\frac{4}{n^2}i^2\right]\left(\frac{2}{n}\right)$$
$$= \sum_{i=1}^{n}\left[\frac{4}{n}+\frac{8}{n^2}i-\frac{8}{n^3}i^2\right]$$

$$A = \lim_{n\to\infty}\left[\frac{4}{n}(n) + \frac{8}{n^2}\frac{n(n+1)}{2} - \frac{8}{n^3}\frac{n(n+1)(2n+1)}{6}\right]$$
$$= 4 + 4 - \frac{8}{3} = \frac{16}{3}$$

18. (a) $y = 8.79x^2 - 6.2x - 0.4$

(b) Velocity = Derivative = $17.58x - 6.2$

At $x = 5$, velocity ≈ 81.7 ft/sec.

Chapters 10–11 Cumulative Test Solutions

1. $(-6, 1, 3)$

2. $(0, -4, 0)$

3. $d = \sqrt{(4-(-2))^2 + (-5-3)^2 + (1-(-6))^2}$

$= \sqrt{36 + 64 + 49}$

$= \sqrt{149}$

4. $d_1 = 3, d_2 = 4, d_3 = \sqrt{4^2 + 3^2} = 5$

$d_1^2 + d_2^2 = d_3^2$

5. Midpoint: $\left(\frac{3-5}{2}, \frac{4+0}{2}, \frac{-1+2}{2}\right) = \left(-1, 2, \frac{1}{2}\right)$

6. Center $= (2, 2, 4)$

Radius $= \sqrt{2^2 + 2^2 + 4^2} = \sqrt{24}$

$(x-2)^2 + (y-2)^2 + (z-4)^2 = 24$

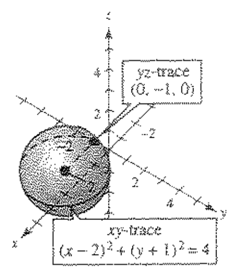
7. xy-trace: $(z = 0)$

$(x-2)^2 + (y+1)^2 = 4$, Circle

yz-trace: $(x = 0)$

$4 + (y+1)^2 + z^2 = 4$ or $(y+1)^2 + z^2 = 0$, Point

$(0, -1, 0)$, Point



8. $\mathbf{u} \cdot \mathbf{v} = \langle 2, -6, 0\rangle \cdot \langle -4, 5, 3\rangle$

$= -8 - 30 = -38$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -6 & 0 \\ -4 & 5 & 3 \end{vmatrix} = \langle -18, -6, -14\rangle$

9. $\mathbf{u} \cdot \mathbf{v} \neq 0, \mathbf{u} \neq c\mathbf{v} \implies$ neither

10. $\mathbf{u} \cdot \mathbf{v} = -8 - 12 + 20 = 0 \implies$ orthogonal

11. $3\mathbf{u} = \langle -3, 18, -9\rangle = -\mathbf{v} \implies$ parallel

12. $\overrightarrow{DA} = \langle 0, 2, 0\rangle, \overrightarrow{DC} = \langle 2, 1, 0\rangle, \overrightarrow{DH} = \langle 0, 0, 3\rangle$

$\begin{vmatrix} 0 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 12$ cubic units

13. (a) Vector is $\langle 5+2, 8-3, 25-0\rangle = \langle 7, 5, 25\rangle$.

$x = -2 + 7t, y = 3 + 5t, z = 25t$

(b) $\frac{x+2}{7} = \frac{y-3}{5} = \frac{z}{25}$

14. $\mathbf{v} = \langle 2, -4, 1\rangle$ and $P = (-1, 2, 0)$

$x = -1 + 2t$

$y = 2 - 4t$

$z = t$

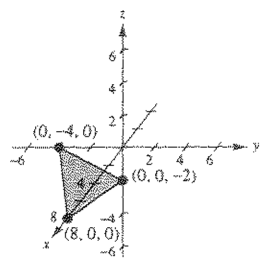
15. $\mathbf{u} = \langle -2, 3, 0\rangle, \mathbf{v} = \langle 5, 8, 25\rangle$

$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 0 \\ 5 & 8 & 25 \end{vmatrix} = \langle 75, 50, -31\rangle$

Normal to plane

Plane: $75x + 50y - 31z = 0$

16.



17. $\mathbf{n} = \langle 2, -5, 1 \rangle$, $Q = (0, 0, 25)$, $P = (0, 0, 10)$ in plane, $\overrightarrow{PQ} = \langle 0, 0, 15 \rangle$

$$D = \frac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \frac{15}{\sqrt{30}} = \frac{\sqrt{30}}{2} \approx 2.74$$

18. Normal to plane containing: $(-1, -1, 3)$, $(0, 0, 0)$ and $(2, 0, 0)$ is

$$\langle -1, -1, 3 \rangle \times \langle 2, 0, 0 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -1 & 3 \\ 2 & 0 & 0 \end{vmatrix} = \langle 0, 6, 2 \rangle \text{ or } \mathbf{n}_1 = \langle 0, 3, 1 \rangle$$

Normal to front face is: $\langle 1, -1, 3 \rangle \times \langle 0, 2, 0 \rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 3 \\ 0 & 2 & 0 \end{vmatrix} = \langle -6, 0, 2 \rangle$ or $\mathbf{n}_2 = \langle -3, 0, 1 \rangle$

Angle between sides: $\cos\theta = \dfrac{|\mathbf{n}_1 \cdot \mathbf{n}_2|}{\|\mathbf{n}_1\|\,\|\mathbf{n}_2\|} = \dfrac{1}{\sqrt{10}\sqrt{10}} = \dfrac{1}{10} \implies \theta \approx 84.26^\circ$

19. $\displaystyle\lim_{x\to 4} (5x - x^2) = 5(4) - 4^2 = 4$

20. $\displaystyle\lim_{x\to -2^+} \frac{x+2}{(x+2)(x-1)} = \lim_{x\to -2^+} \frac{1}{x-1} = -\frac{1}{3}$

21. $\displaystyle\lim_{x\to 7} \frac{x-7}{(x-7)(x+7)} = \lim_{x\to 7} \frac{1}{x+7} = \frac{1}{14}$

22. $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x} \cdot \frac{\sqrt{x+4}+2}{\sqrt{x+4}+2} = \lim_{x\to 0} \frac{(x+4)-4}{x\left(\sqrt{x+4}+2\right)} = \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{2+2} = \frac{1}{4}$

23. $\displaystyle\lim_{x\to 4^-} \frac{|x-4|}{x-4} = -1$

24. $\displaystyle\lim_{x\to 0} \sin\left(\frac{\pi}{x}\right)$ does not exist.

25. $\displaystyle\lim_{x\to 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x} = \lim_{x\to 0} \frac{2-(x+2)}{x(x+2)2}$

$$= \lim_{x\to 0} \frac{-1}{(x+2)2} = -\frac{1}{4}$$

26. $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x\to 0} \frac{(x+1)-1}{x\left[\sqrt{x+1}+1\right]}$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{2}$$

27. $\displaystyle\lim_{x\to 2^-} \frac{x-2}{x^2-4} = \lim_{x\to 2^-} \frac{x-2}{(x-2)(x+2)}$

$$= \lim_{x\to 2^-} \frac{1}{x+2} = \frac{1}{4}$$

28. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{4 - (x+h)^2 - (4 - x^2)}{h}$

$= \lim_{h \to 0} \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$

$= \lim_{h \to 0} \frac{-2xh - h^2}{h}$

$= \lim_{h \to 0} (-2x - h) = -2x$, Slope

At $(0, 4)$, $m = 0$.

29. $f(x) = \sqrt{x+3}$

$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}$

$= \lim_{h \to 0} \frac{(x+h+3) - (x+3)}{h\left[\sqrt{x+h+3} + \sqrt{x+3}\right]}$

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} = \frac{1}{2\sqrt{x+3}}$

At $(-2, 1)$, $m = \frac{1}{2}$.

30. $f(x) = \frac{1}{x+3}$

$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{x+h+3} - \frac{1}{x+3}}{h}$

$= \lim_{h \to 0} \frac{(x+3) - (x+h+3)}{h(x+h+3)(x+3)}$

$= \lim_{h \to 0} \frac{-1}{(x+h+3)(x+3)}$

$= \frac{-1}{(x+3)^2}$

At $\left(1, \frac{1}{4}\right)$, $m = -\frac{1}{16}$.

31. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - h}{h}$

$= \lim_{h \to 0} (2x + h - 1) = 2x - 1$, Slope

At $(1, 0)$, $m = 1$.

32. $\lim_{x \to \infty} \frac{x^3}{x^2 - 9}$ does not exist.

Function increases without bound.

33. $\lim_{x \to \infty} \frac{3 - 7x}{x + 4} = -7$

34. $\lim_{x \to \infty} \frac{3x^2 + 1}{x^2 + 4} = 3$

35. $\lim_{x\to\infty} \dfrac{2x}{x^2 + 3x - 2} = 0$

36. $\lim_{x\to\infty} \dfrac{3 - x}{x^2 + 1} = 0$

37. $\lim_{x\to\infty} \dfrac{3 + 4x - x^3}{2x^2 + 3}$

does not exist. Function decreases without bound.

38. $\sum_{i=1}^{50}(1 - i^2) = 50 - \dfrac{50(51)(101)}{6} = -42{,}875$

39. $\sum_{k=1}^{20} (3k^2 - 2k) = 3\dfrac{20(21)(41)}{6} - 2\dfrac{20(21)}{2}$

$= 8610 - 420 = 8190$

40. $\sum_{i=1}^{40} (12 + i^3) = 12(40) + \dfrac{40^2(41)^2}{4}$

$= 480 + 672{,}400 = 672{,}880$

41. Area $\approx \dfrac{1}{2}[1 + 2 + 3 + 4 + 5 + 6]$

$= \dfrac{21}{2}$ square units

42. Area $\approx \frac{1}{2}[4.875 + 4.5 + 3.875 + 3]$

$= 8.125$ square units

43. Area $\approx \frac{1}{2}\left[\frac{9}{16} + 1 + \frac{25}{16} + \frac{9}{4}\right] = \frac{43}{16} = 2.6875$

44. Area $\approx \dfrac{1}{4}\left[\dfrac{1}{1 + \left(-\frac{3}{4}\right)^2} + \dfrac{1}{1 + \left(-\frac{1}{2}\right)^2} + \dfrac{1}{1 + \left(-\frac{1}{4}\right)^2} + \dfrac{1}{1 + 0} + \dfrac{1}{1 + \left(\frac{1}{4}\right)^2} + \dfrac{1}{1 + \left(\frac{1}{2}\right)^2} + \dfrac{1}{1 + \left(\frac{3}{4}\right)^2} + \dfrac{1}{1 + 1^2}\right]$

$= \dfrac{1}{4}\left[2(0.64) + 2(0.8) + 2(0.941176) + 1 + \dfrac{1}{2}\right]$

≈ 1.566 square units

45. $f(x) = x + 2,\ [0, 1]$

The width of each rectangle is $1/n$. The height is

$f\left(\dfrac{i}{n}\right) = \dfrac{i}{n} + 2.$

$A \approx \sum_{i=1}^{n}\left[\dfrac{i}{n} + 2\right]\dfrac{1}{n} = \sum_{i=1}^{n}\left(\dfrac{1}{n^2}i + \dfrac{2}{n}\right)$

$A = \lim_{n\to\infty}\left[\dfrac{1}{n^2}\dfrac{n(n + 1)}{2} + \dfrac{2}{n}(n)\right]$

$= \dfrac{1}{2} + 2 = \dfrac{5}{2}$

46. $f(x) = 8 - 2x,\ [-4, 4]$

The width of each rectangle is $8/n$. The height is

$f\left(-4 + \dfrac{8i}{n}\right) = 8 - 2\left(-4 + \dfrac{8i}{n}\right) = 16 - \dfrac{16i}{n}.$

$A \approx \sum_{i=1}^{n}\left(16 - \dfrac{16i}{n}\right)\left(\dfrac{8}{n}\right)$

$= \sum_{i=1}^{n}\left(\dfrac{128}{n} - \dfrac{128i}{n^2}\right)$

$A = \lim_{n\to\infty}\left[\dfrac{128}{n}(n) - \dfrac{128}{n^2}\dfrac{n(n + 1)}{2}\right]$

$= 128 - 64 = 64$

47. $f(x) = 2x + 5,\ [-1, 3]$

The width of each rectangle is $4/n$. The height is

$$f\left(-1 + \frac{4i}{n}\right) = 2\left(-1 + \frac{4i}{n}\right) + 5 = 3 + \frac{8i}{n}.$$

$$A \approx \sum_{i=1}^{n}\left(3 + \frac{8i}{n}\right)\left(\frac{4}{n}\right) = \sum_{i=1}^{n}\left(\frac{12}{n} + \frac{32}{n^2}i\right)$$

$$A = \lim_{n\to\infty}\left[\frac{12}{n}(n) + \frac{32}{n^2}\frac{n(n+1)}{2}\right]$$

$$= 12 + 16 = 28$$

48. $f(x) = x^2 + 1, [0, 4]$

The width of each rectangle is $4/n$. The height is

$$f\left(\frac{4i}{n}\right) = \left(\frac{4i}{n}\right)^2 + 1.$$

$$A \approx \sum_{i=1}^{n}\left(\frac{16i^2}{n^2} + 1\right)\left(\frac{4}{n}\right) = \sum_{i=1}^{n}\left(\frac{64}{n^3}i^2 + \frac{4}{n}\right)$$

$$A = \lim_{n\to\infty}\left[\frac{64}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{4}{n}(n)\right]$$

$$= \frac{64}{3} + 4 = \frac{76}{3}$$

49. $f(x) = 4 - x^2,\ [0, 2]$

The width of each rectangle is $2/n$. The height is

$$f\left(\frac{2i}{n}\right) = 4 - \left(\frac{2i}{n}\right)^2.$$

$$A \approx \sum_{i=1}^{n}\left[4 - \frac{4i^2}{n^2}\right]\left(\frac{2}{n}\right) = \sum_{i=1}^{n}\left(\frac{8}{n} - \frac{8i^2}{n^3}\right)$$

$$A = \lim_{n\to\infty}\left[\frac{8}{n}(n) - \frac{8}{n^3}\frac{n(n+1)(2n+1)}{6}\right]$$

$$= 8 - \frac{8}{3} = \frac{16}{3}$$

50. Width: $\frac{1}{n}$, Height: $f\left(\frac{i}{n}\right) = 1 - \left(\frac{i}{n}\right)^3$

$$A \approx \sum_{i=1}^{n}\left(1 - \left(\frac{i}{n}\right)^3\right)\left(\frac{1}{n}\right) = \frac{1}{n}\sum_{i=1}^{n} 1 - \frac{1}{n^4}\sum_{i=1}^{n} i^3$$

$$= \frac{1}{n}(n) - \frac{1}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$$

$$A = \lim_{n\to\infty}\left[1 - \frac{1}{n^4}\left(\frac{n^2(n+1)^2}{4}\right)\right] = 1 - \frac{1}{4} = \frac{3}{4}$$